

**Psychology 453: Origins of Numbers**  
**Fall 2009**

**Course Location :** Hagey Hall 373  
**Meeting Times:** Wednesdays 10:30am to 12:30pm

**Instructor:** Mathieu Le Corre  
**Office address:** PAS 4010  
**Office Hours:** By appointment  
**E-mail:** [mlecorre@uwaterloo.ca](mailto:mlecorre@uwaterloo.ca) or via ACE.

**COURSE DESCRIPTION**

This course will take a multi-disciplinary approach to the study of the nature and acquisition of knowledge of the natural numbers and arithmetic. We will be reading and discussing both seminal and state-of-the-art research on: the nature of numerical representations in human infants and in non-human animals, the neural substrates of basic numerical representations, the construction of new numerical knowledge, variations in arithmetic systems across human cultures, and on the relationship (if any) between potentially innate individual differences in numerical representations and individual differences in mathematical knowledge and skills learned in school.

**COURSE OBJECTIVES**

My primary goals are two-fold. First, readings and discussions of research on the acquisition of numerical knowledge and concepts will be used as a means of engaging you in learning about the central questions that arise in the study of cognitive development and about the multiple methodological approaches that can be used to answer these questions. In particular, I hope that you will come to appreciate that *any* explanation of learning (in humans or in animals) must have three parts: (1) a characterization of the concepts or knowledge to be acquired; (2) a characterization of the organism's concepts and knowledge prior to the acquisition of the new knowledge; and (3) a description of how the organism uses the knowledge at its disposal to acquire the new knowledge.

Second, I hope to help you improve your skill at decomposing, synthesizing and appraising scientific arguments. The course will provide two major "playing fields" for working on these skills: group discussions and written reports. In all of these exercises, I will invite you to find not only what's wrong with the papers you read, but also what contribution the authors have made to their field, whether it be a new piece of knowledge – however small – or even just a new way of looking at a problem. In light of this goal, I will encourage you to show that you understand (or at least have really tried to understand) the arguments in the papers they read before you criticize them.

## ASSIGNMENTS AND GRADING

**Weekly rapid reactions (15%).** Each week, you will be asked to post your comments, criticisms, or praise for one or more of the readings of that week on the course website. Your comments need not be longer than some 250 words (about 1 paragraph), though they can be longer if you have a lot to say. **Each week, you will find a folder with a discussion forum under the “lessons” tab of the course website. Your comments must be posted on the discussion forum by 5pm on the day before class. Late comments will not be accepted. Failure to post a comment will lead to the deduction of 2% from your final grade (up to a maximum of 15%). On the weeks when you turn in a short summary and commentary (see below), you will not be expected to post a rapid reaction.**

**All you need to do to get your full 15% is post each week.** You should not need to spend more than 45 minutes a week to write these. That’s why I call them “rapid reactions”; they don’t need to be completely well-formed, multi-part arguments. My only requirement is that your comments show that you’ve read the paper(s) you’re talking about (and trust me, I can tell). **Posting a rapid reaction that’s too vaguely related to the papers of the week will be treated like a failure to post a comment; i.e. it will lead to the deduction of 2% from your final grade.** Grades aside, you should also know that you can make my day if you show me that you’ve really been thinking about the readings. The range of possible comments is about as wide as your imagination. For example, you could:

- point out a methodological flaw in an experiment (please don’t say that the sample size was too small... no paper with such basic methodological flaws gets published).
- explain how the paper you read contrasts with/agrees with/amplifies/invalidates something else we’ve talked about
- say how a paper led you to change the way you thought about something
- say how the theory and/or results of a paper conflict with common sense or received wisdom
- suggest an alternative interpretation of some experimental result(s)
- suggest an explanation that resolves a conflict between different experimental results
- point out a contradiction or particularly obscure passage in some author(s)’ argument
- relate a phenomenon described in a paper to something you have observed in your own life or learned about in some other context
- compare two distinct papers that talk about a similar problem and say which one is stronger
- express your frustration at not being able to understand a part of a reading (but do tell us your best guess(es) about what the authors could be talking about and make sure that we can understand what it is that you can’t understand...)
- etc...

### **Short summaries and commentaries (30%).**

This assignment has two parts. First, you must choose one of the readings assigned for a given week and summarize it in 300 words or less. Your summary include descriptions of: (1) the main question(s) addressed in the paper and/or the authors' hypotheses; (2) the experiment(s) used to answer these questions; (3) the results of these experiments; (4) the implications of the results for the hypotheses tested. The second part is just like a rapid reaction but longer; i.e. you should write a 500 word (about 1.5 pages of double-spaced text) commentary on the paper you have summarized.

**You will have to turn in at least two short summaries.** You will find a drop box for short summaries and commentaries in each week's folder under the "lessons" tab. Turn in your short summaries and commentaries by uploading a Word file with your assignment in the drop box. **Like rapid reactions, they will be due by 5pm on the day before class. Each of them will be worth 15%. Which papers you choose to summarize and comment on is up to you. You can choose to turn in three summaries and short comments and I will grade your two best.** I highly suggest you turn in three so you can get clear feedback about exactly what I expect and about how you are doing. **On the weeks when you turn in a short summary and commentary, you will not be expected to post a rapid reaction.**

### **Final paper (55%).**

This assignment has three parts: a bibliography (10%); a first draft (15%); and a final paper (30%). **All deadlines for this set of assignments are posted on the course calendar on the course website. You will start off this process by having an individual meeting with me in the week of November 2<sup>nd</sup> to November 6<sup>th</sup>.** At this meeting, you will tell me what you are interested in writing about (topic and framework – see below), and I will suggest a few references to get you started on your research. We will establish the schedule of these meetings in class on **October 28<sup>th</sup>.**

Since the bibliography is a means of getting to the first draft, and the first draft is a means of getting to a good final paper, these assignments are described in reverse order.

**Final paper. Your final paper will be due on FRIDAY, DECEMBER 11<sup>th</sup> by 5pm. THERE WILL BE A PENALTY FOR LATE PAPERS: 5% will be taken off the grade for your paper for each day between the deadline and the time you turned it in. You will turn in your paper by uploading it in a drop box on the course website.**

Your paper can be on any topic in linguistic and/or cognitive development. Regardless of the topic you choose, your paper must fall within at least one of the following four frameworks. Note that the frameworks are not mutually exclusive. Your paper may touch on more than one, but it must at least fully cover one of the frameworks.

### Framework 1: Building blocks

- a. Provide a clear description of the knowledge, concepts or cognitive/linguistic structure to be acquired.
- b. Provide a description of the (potential) building blocks of the knowledge, concepts, or cognitive structure to be acquired. This description must be woven out of the results of experimental research. Building blocks can be inferred from:
  - a. Research on babies and/or preschoolers
  - b. Research on how children acquire the language/symbols that express the knowledge to be acquired
  - c. Cross-cultural research
  - d. Research on the nature of related knowledge/concepts in other animals.

### Framework 2: Learning mechanisms

- a. Provide a clear description of the knowledge, concepts or cognitive/linguistic structure to be acquired.
- b. Provide a description of the (potential) building blocks of the knowledge, concepts, or cognitive structure to be acquired. This description must be woven out of the results of experimental research.
- c. Review theories and evidence (e.g. training studies) concerning the learning mechanism whereby learners acquire the full knowledge.

Note: The only difference between this framework and framework 1: building blocks is that the emphasis here is more on research on the learning mechanism than on research on the building blocks.

### Framework 3: Language and thought

- a. Identify a domain of perception or cognition (e.g. color perception, or spatial reasoning).
- b. Describe at least two languages that vary in how they structure this domain of perception or cognition (e.g. one language has 50 color terms, another only has color terms for red, green, and blue).
- c. Review evidence as to whether the differences in linguistic structure cause differences in perception or cognition (e.g. speakers of the language of 50 color terms are better at remembering colors than speakers of the language with 3 terms).

### Framework 4: Comparative psychology

- a. Identify a domain of perception or cognition
- b. Review evidence comparing the structure of this domain of perception or cognition in humans and in non-human animals. Be clear about similarities and differences across species.

Within these general frameworks, there are two possible types of paper:

1. *Integrative review.* Write an integrative review of **at least six different sources** on your topic within your chosen framework(s). The sources can converge on the same description of your topic from different methods, or they may disagree. If the sources converge, clearly explain what each source contributes and how the different pieces fit together. If the sources disagree, make a coherent argument as to what you think is the best interpretation. You can defend an interpretation of your own if you are not satisfied with any of those available in the literature you reviewed.
2. *Experimental proposal.* Review **at least three different sources** on your topic, identify gaps, flaws, or new questions in the research you reviewed, and propose an experiment that could fill the gap, fix the flaw, or answer the new question. Your experimental proposal must include a clear description of the methods you would use, and of what different outcomes would mean.

**When you turn in your final paper, include a separate page that states:**

1. The title of your paper;
2. The domain of language, cognition, or perception you wrote about (e.g. color perception, understanding of mechanics, theory of mind, etc...);
3. The framework(s) you have chosen;
4. Whether your paper is an integrative review or an experimental proposal;

**The separate page should also include a 250-word abstract that summarizes your paper.**

*First draft.* **Turn in a hard copy of your first draft in class on Wednesday, November 25<sup>th</sup>.** Your first draft should be a complete draft of the final paper. So the instructions for it are the same as for the final draft. The purpose of this draft is to get you writing more than 24 hours before the deadline for the final paper and to give you an opportunity to get feedback on your writing and thinking. So, your grade for this assignment will be minimally influenced by the quality of your writing and thinking (3 out of 15 points). You can get the rest of the marks (12/15) if your paper:

1. Clearly fits within one or more of the four frameworks;
2. If it is an integrative review, has at least six sources and thoroughly reviews all of them;
3. If it is an experimental proposal, clearly reviews at least three sources and has a complete experimental proposal.

In other words, when you get this assignment back, you should pay more attention to my comments than to the grade you got. That being said, you will get a lot more out of this and you will probably write a better final paper if you give the first draft your best shot. I'm also more likely to see that your paper fits within the framework(s) you have chosen if it is well written.

*Bibliography.* **Your bibliography is due in class on November 11<sup>th</sup>. Please turn in a hard copy.** Your bibliography will list complete references of the sources you are proposing to use, as well as 3 or 4 sentences explaining the relevance of each source to your paper. If you are writing an **integrative review**, you will need **at least six references**. If you are writing an **experimental proposal**, you will need **at least three sources**. Moreover, you will need to write 3 or 4 sentences about the problem or question you will try to address with your experiment.

## **POLICY ON ACADEMIC INTEGRITY**

In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility

**Grievance:** A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70 - Student Petitions and Grievances, Section 4, <http://www.adm.uwaterloo.ca/infosec/Policies/policy70.htm>

**Discipline:** A student is expected to know what constitutes academic integrity, to avoid committing academic offenses, and to take responsibility for his/her actions. A student who is unsure whether an action constitutes an offense, or who needs help in learning how to avoid offenses (e.g., plagiarism, cheating) or about 'rules' for group work/collaboration should seek guidance from the course professor, academic advisor, or the Undergraduate Associate Dean. When misconduct has been found to have occurred, disciplinary penalties will be imposed under Policy 71 - Student Discipline. For information on categories of offenses and types of penalties, students should refer to Policy 71 - Student Discipline, <http://www.adm.uwaterloo.ca/infosec/Policies/policy71.htm>

**Appeals:** A student may appeal the finding and/or penalty in a decision made under Policy 70 - Student Petitions and Grievances (other than regarding a petition) or Policy 71 - Student Discipline if a ground for an appeal can be established. Read Policy 72 - Student Appeals, <http://www.adm.uwaterloo.ca/infosec/Policies/policy72.htm>

## **POLICY ON ABSENCES**

Students who are requesting accommodation for course requirements (assignments, midterm tests, final exams, etc.) due to illness should do the following:

- seek medical treatment as soon as possible and obtain a completed UW Verification of Illness Form:  
[http://www.healthservices.uwaterloo.ca/Health\\_Services/verification.html](http://www.healthservices.uwaterloo.ca/Health_Services/verification.html)
- submit that form to the instructor within 48 hours.

- (preferably) inform the instructor by the due date for the course requirement that you will be unable to meet the deadline and that documentation will be forthcoming.

In the case of a missed assignment deadline or midterm test, the instructor will either

- a) waive the course component and re-weight remaining term work as he/she deems fit according to circumstances and the goals of the course, or
- b) provide an extension.

## **SCHEDULE OF TOPICS AND READINGS.**

**Wednesday, September 16<sup>th</sup>. Introduction.**

**Wednesday, September 23<sup>rd</sup>. Number in Human Babies. Part I: Analog magnitudes.**

Dehaene, S. (1997). The Number Sense.

- Chapter 1: “Talented & Gifted Animals” (pp. 13-34).
  - Focus on “The Accumulator Metaphor” and browse through the rest.

Lipton, J. & Spelke, E.S. (2004). Discrimination of Large and Small Numerosities by Human Infants. *Infancy*, 5(3), 271-290.

McCrink, C. & Wynn, K. (2004). Large Number Addition and Subtraction by 9-month-old Infants. *Psychological Science*, 15(11), 776-781.

**Wednesday, September 30<sup>th</sup>. Number in Human Babies. Part II: Objects & Number.**

Feigenson, L, Carey, S., & Hauser, M., (2002). The Representations Underlying Infants’ Choice of More. *Psychological Science*, 13, 150-156.

Feigenson, L., & Carey, S. (2003). Tracking Individuals via Object-Files: Evidence from Infants’ Manual Search. *Developmental Science*, 6(5), 568-584.

Izard, V., Dehaene-Lamberts, G. & Dehaene, S. Distinct Cerebral Pathways for Object Identity and Number in Human Infants. *PLOS Biology*, 6(2).

**Wednesday, October 7<sup>th</sup>. The evolutionary history of arithmetic: Number in non-human primates.**

Brannon, E.M., & Terrace, H.S. (1998). Ordering of the numerosities 1-9 by monkeys. *Science*, 282(5389), 746-749.

Hauser, M., Carey, S. & Hauser, L. (2000) Spontaneous number representation in semi-free ranging rhesus monkeys. *Proceedings of the Royal Society B: Biological Sciences*, 267, 829-833.

Matsuzawa, T. (1985). Use of numbers by a chimpanzee. *Nature*, 315, 57-59.

**Wednesday, October 14<sup>th</sup>. Counting: innate or constructed? (Part I).**

**Read 1<sup>st</sup>**

Gelman, R. (1982). Basic numerical abilities. In R. J. Sternberg (Ed.), *Advances in the psychology of human intelligence*. Hillsdale, N. J.: Erlbaum. Vol. 1., (4) 181-205.

**Read 2<sup>nd</sup>**

Gallistel, C.R. & Gelman, R. (1992). Preverbal and verbal counting and computation.

- “The Preverbal counting model” (pp. 51-52).
- “Implications for adult numerical competence” (pp. 55-58 up to but not including “Judgments of digit order”).
- “The acquisition of verbal counting” (p. 65).

**Read 3<sup>rd</sup>**

Le Corre, M., Van de Walle, G., Brannon, E.M. & Carey, S. (2006). Re-visiting the Performance/Competence Debate in the Acquisition of the Counting Principles. *Cognitive Psychology*.

**Wednesday, October 21<sup>st</sup>. “Bootstrapping”: a mechanism for constructing knowledge of counting out of representations that are not count-like?**

Fodor, J. (1983). On the Impossibility of Acquiring More Powerful Structures. In Piatelli-Palmarini, M. (Ed.), *The Debate between Jean Piaget and Noam Chomsky* (pp. 142-162).

\*\*Focus on Fodor’s target article, but do read the commentaries & discussions and pay particular attention to Piaget’s intervention and Fodor’s response.

Carey, S. (2009). Beyond Core Cognition: Natural Number. In *The Origin of Concepts* (pp. 287-334).

**Wednesday, October 28<sup>th</sup>**. Number, sets, and one-to-one correspondence. Why so late?

Flavell, J. (1963). The Developmental Psychology of Jean Piaget.

- “The Algebra of Groups” (pp. 135-136).

Piaget, J. (1952/1965). The Child’s Conception of Number.

- “Spontaneous Correspondence – Cardinal Value of Sets”. (pp. 65-85 in my 1965 edition).

Gelman, R. (1982). Accessing one-to-one correspondence: Still another paper about conservation. *British Journal of Psychology*, 73, 209-220.

**Wednesday, November 4<sup>th</sup>**. Number: neural substrates.

Dehaene, S., Spelke, E., Pinel, P., Stanescu, R., & Tsivkin, S. (1999). Sources of mathematical thinking: Behavioral and brain-imaging evidence. *Science*, 284, 970-974.

Lemer, C., Dehaene, S., Spelke, E., & Cohen, L. (2003). Approximate quantities and exact number words: Dissociable systems. *Neuropsychologia*, 41, 1942-1958.

Holloway, I.D., Price, G.R. & Ansari, D. (in press) Common and segregated neural pathways for the processing of symbolic and nonsymbolic numerical magnitude: an fMRI study. *NeuroImage*.

**Wednesday, November 11<sup>th</sup>**. Relations between basic numerical competence and individual differences in mathematical achievement.

Booth, J.L, & Siegler, R.S. (2008). Numerical magnitude representations influence arithmetic learning. *Child Development*. 79(4), 1016-1031.

Holloway, I.D. & Ansari, D. (2009) Mapping numerical magnitudes onto symbols: The numerical distance effect and individual differences in children’s math achievement. *Journal of Experimental Child Psychology*, 103, 17-29.

Halberda, J., Mazocco, M.M.M., Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with maths achievement. *Nature*, 455, 665-668.

**Wednesday, November 18<sup>th</sup>. Number across cultures.**

Saxe, G. (1981). Body parts as numerals: A developmental analysis of numeration among the Oksapmin in Papua New Guinea. *Child Development*, 52, 306-316.

Frank, M.C., Everett, D.L., Fedorenko, E., & Gibson, E. (2008). Number as a cognitive technology: evidence from Piraha language and cognition. *Cognition*, 108, 819-824.

**Wednesday, November 25<sup>th</sup>. Children invent mathematics.**

Hughes, M. (1986). *Children and Number: Difficulties in Learning Mathematics*.

- Chapter 5. Children's invention of written arithmetic (pp. 53-78).

Nunes, T., Schliemann, A.-T., Caraher, D. (1993). *Street Mathematics and School Mathematics*.

- Chapter 2. Arithmetic in the street and in schools (pp. 13-27).

**Wednesday, December 2<sup>nd</sup>. Why there are so few women academics in the natural sciences & in math: some ideas from former Harvard president Larry Summers, from the press, and from a few empirical studies.**

**The president**

Summers, L. (2005, January 14). Remarks at the NBER Conference on Diversifying the Science & Engineering Workforce.

**The press (Read one of the two)**

Associated Press (2005, February 27). Some experts back Harvard Prez's remarks.

Pollitt, K. (2005, February 21) Summers of our discontent. *The Nation*. Available at: [www.thenation.com/doc/20050221/pollitt](http://www.thenation.com/doc/20050221/pollitt)

**The science**

Geary, D.C. (1996). Sexual selection and sex differences in mathematical abilities. *Behavioral and Brain Sciences*, 19(2), 229-284.

Spelke, E. S. (2005). Sex differences in intrinsic aptitudes in mathematics and science? A critical review. *American Psychologist*, 60(9), 950-958.

Optional (but highly recommended!!!): Read or watch the "Science of Gender and Science" debate between Harvard psychology professors Steven Pinker and Liz Spelke at: [www.edge.org/3rd\\_culture/debate05/debate05\\_index.html](http://www.edge.org/3rd_culture/debate05/debate05_index.html)