

Course Announcement Winter 2017  
PMATH 945 Topics in Algebra: **Valued Fields**  
MWF 10:30–11:20 MC 5479  
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This will be a graduate level algebra course that may also interest students of number theory, algebraic geometry, and logic, though these latter subjects will not be explicitly addressed. The only prerequisites will be a thorough background in abstract algebra: groups, rings, fields, some Galois theory, basic point-set topology. I expect to follow the book Valued Fields by Engler and Prestel (Springer 2005), copies of which will be carried by the bookstore, held on reserve in the library, and also available electronically through the library. Students will be evaluated on the basis of approximately six assignments and an oral final exam.<sup>1</sup>

**Abstract.** Besides the usual (archimedean) absolute value on  $\mathbb{Q}$ , there exists, for every prime  $p$ , the absolute value which assigns to  $p$  the value  $\frac{1}{e}$  and to all other primes the value 1. The completion of  $\mathbb{Q}$  with respect to this absolute value is the field  $\mathbb{Q}_p$  of  $p$ -adic numbers. The  $p$ -adics play an important role in number theory. In the nineteen thirties Krull introduced *valuation* as a generalisation of absolute value wherein the values land in an arbitrary ordered abelian group (rather than the positive reals). Valued fields and their *henselisations* (extensions in which Hensel's Lemma holds, these play the role of completions) continue to have significant applications in number theory and algebraic geometry. This will be a course on the theory of valued fields, including an introduction to the  $p$ -adics and culminating in a discussion of the work of Ax and Kochen on  *$p$ -adically closed* fields if time permits.

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<sup>1</sup>This is my current intention, it is of course subject to change.