

# Pure Math 445/745

## Bilinear Forms and Group Representations

**Class times and place:** TTh from 8:30–9:50AM in ML 349

**Instructor:** Ben Webster

**Office:** MC 5429

**Office Hours:** Tuesday 10–11AM, Thursday 2–3PM

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**Website:** <https://learn.uwaterloo.ca>

You are expected to regularly read your UWaterloo email and visit the course website on LEARN for announcements.

### Description

The course introduces students to the representation theory of finite groups. It begins with a discussion of the elementary properties of linear representations of finite groups, subrepresentations, irreducibility, and various ways of combining two representations to get another: direct sums, tensor (Kronecker) products, and symmetric squares. Next comes character theory, including the statement and proof of Schur's Lemma, the orthogonality relations, and the canonical decomposition of a representation. Together with the notion of an induced representation, the course now has the tools to explore a variety of examples. The course then turns to explore the connections between the representations of  $G$  and modules over the group ring  $C[G]$ . Properties of the group ring in particular, and semisimple rings more generally, are explored; including the fact that every semisimple ring is a product of matrix algebras over division rings (the Artin-Wedderburn theorem). Applications of the structure of  $C[G]$  to the representation theory of  $G$  are discussed. The final part of the course studies induced representations in greater detail, applying the tools developed so far to prove the Frobenius reciprocity theorem and Mackey's criterion for irreducibility. Time permitting, we apply the above theorems to investigate further particular examples of induced representations.

### Texts

The primary text for the class will be "Linear Representations of Finite Groups" by Jean-Pierre Serre. I suggest having access to a physical or electronic copy of this book. I may suggest readings from other online notes such as those of [Teleman](#) and [Webb](#).

### Grades and assignments

Grades will be based on individual homework assignments (40%) and a final exam (60%).

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