

PMATH 446/646 – Winter 2019
Introduction to Commutative Algebra

MWF 10:30–11:20 PHY 313 Instructor: Rahim Moosa

This course is primarily targeted at Pure Math honours students that wish to further their knowledge in algebra. It is of particular importance to students interested in algebraic geometry and algebraic number theory.

PMATH 347 (Groups and Rings) is a prerequisite and PMATH 348 (Fields and Galois Theory) is a corequisite.

The textbook for the course will be Introduction to Commutative Algebra by Atiyah and Macdonald. A courseware package of this book will be produced and sold at Media.doc.

Outline. The first part of this course is a careful introduction to the theory of modules over a commutative unitary ring. The relevant ring and ideal theory is reviewed as needed. The following foundational notions are discussed: submodule, quotient, sum and product, finitely generated module, exact sequence, tensor product, flatness, R-algebra, direct limit, localisation, chain conditions. The classification of finitely generated modules over principal ideal domains is proved. The second part of the course focuses on integral extensions and integrally closed domains. The main results here are the “going up” and “going down” theorems. Noether normalisation and Hilbert Nullstellensatz are also done in this section. The final core part of the course is the existence and uniqueness of primary decomposition for ideals in Noetherian rings. There should be approximately two weeks at the end of the course to pursue additional topics. Possibilities include connections to algebraic geometry (e.g., the geometric significance of earlier topics, Krull dimension) and algebraic number theory (e.g., DVRs, Dedekind domains).

Assessment. Approximately 6 homework assignments totaling 30% and a final exam worth 70%.