Course Description:
General measures, measurability, Caratheodory extension theorem and construction of measures, integration theory, convergence theorems, $L^p$-spaces, absolute continuity, differentiation of monotone functions, Radon-Nikodym theorem, product measures, Fubini’s theorem, signed measures, Urysohn’s lemma, Riesz Representation theorems for classical Banach spaces.

Prerequisite:
PMATH 450/650 or equivalent (knowledge of Lebesgue measure will be assumed).

Textbook:
*Measure Theory*, Second Edition, by Donald L. Cohn (Birkhäuser), available electronically from the library, is recommended but not required, as my notes will be self-contained. Two other good references are *Real Analysis* by G.B. Folland or H.L. Royden.

Marking Scheme:

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<thead>
<tr>
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<th>PMATH 451</th>
<th>PMATH 651</th>
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<tbody>
<tr>
<td>Assignments</td>
<td>50%</td>
<td>45%</td>
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<tr>
<td>Final Exam</td>
<td>50%</td>
<td>45%</td>
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<tr>
<td>Project</td>
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Assignments:
There will be 5 assignments of equal weight, due Fridays September 26, October 10 and 24, and November 7 and 21, at the beginning of class.

Project:
Graduate students taking PMATH 651 are required to complete a project, consisting of a presentation given during the pre-exam study days of December 2-3, and a written report to be submitted before the presentation.
Group Work:
I expect that you will (and encourage you to) collaborate on the assignments, however the work you submit must be your own. You should work together on the ideas, not the written solutions. I have instructed the grader to alert me to suspiciously similar solutions and cases of plagiarism will be dealt with in the harshest way possible (see the section “Discipline” below).

Late Assignments:
Late assignments will not be accepted except for in the most extreme circumstances.

Academic Integrity:
In order to maintain a culture of academic integrity, member of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. Refer to Academic Integrity website (https://uwaterloo.ca/academic-integrity/) for details.

Grievance:
A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70 (https://uwaterloo.ca/secretariat/policies-procedures-guidelines/policy-70) Student Petitions and Grievances, Section 4. When in doubt, please contact the departments administrative assistant who will provide further assistance.

Discipline:
A student is expected to know what constitutes academic integrity to avoid committing an academic offence, and to take responsibility for his/her actions. A student who is unsure whether an action constitutes an offence, or who needs help in learning how to avoid offences (e.g. plagiarism, cheating) or about rules for group work/collaboration should seek guidance from the course instructor, academic advisor, or the undergraduate Associate Dean. For information on categories of offences and types of penalties, students should refer to Policy 71 (https://uwaterloo.ca/secretariat/policies-procedures-guidelines/policy-71) Student Discipline. For typical penalties check Guidelines for the Assessment of Penalties (https://uwaterloo.ca/secretariat/policies-procedures-guidelines/guidelines/assessment-penalties).

 Appeals:
A decision made or penalty imposed under Policy 70 (Student Petitions and Grievances) (other than a petition) or Policy 71 (Student Discipline) may be appealed if there is a ground. A student who believes he/she has a ground for an appeal should refer to Policy 72 (Student Appeals) www.adm.uwaterloo.ca/infosec/Policies/policy72.htm.

Note for Students with Disabilities:
AccessAbility Services (http://uwaterloo.ca/disability-services/), located in Needles Hall, Room 1132, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with the office at the beginning of each academic term.
Course Outline:

- inadequacy of the Riemann integral
- \( \sigma \)-algebras and measures
- outer measures, pre-measures, Carathéodory and Hahn extension theorems
- Lebesgue and Lebesgue-Stieltjes measure
- regularity and completeness
- measurable functions
- integration
- convergence theorems
- \( L^p \)- and Banach spaces
- Hölder and Minkowski inequalities
- modes of convergence
- signed measures, Hahn and Jordan decomposition theorems
- absolute continuity and the Radon-Nikodym theorem
- singularity and the Lebesgue decomposition theorem
- spaces of measures
- functions of bounded variation, absolutely continuous functions
- Lebesgue differentiation theorem and the Fundamental Theorem of Calculus
- complex measures, total variation
- product measures, Tonelli and Fubini theorems
- Riesz representation theorem(s)