

Outline for the Algebra Comprehensive Examination

The purpose of the graduate comprehensive examination is to test the candidate's knowledge and understanding of basic algebraic topics that include the following:

Linear and Multilinear Algebra:

Jordan and rational canonical form for matrices; Inner product spaces; Spectral theorem; Hermitian, unitary, and normal operators; Properties of positive definite matrices and operators; Dual space and the transpose of an operator; Tensor products of vector spaces.

Group Theory:

Isomorphism Theorems; Jordan-Hölder series; Krull-Schmidt Theorem; Sylow Theorems; Free abelian groups; Structure of finitely generated abelian groups; Divisible abelian groups; Symmetric and Alternating groups; Central series and nilpotent groups; Solvable groups; Free groups; Presentation of groups; Linear groups.

Ring Theory:

Principal ideal domains, Euclidean domains; unique factorization domains; Chinese remainder theorem; Finitely generated modules over PID's; Noetherian and Artinian modules; Polynomial rings and Hilbert's Basis Theorem.

Field Theory:

Algebraic extensions; Algebraic closure; Splitting fields of polynomials; Separable and inseparable extensions; Simple extensions; Galois correspondence; Norms and traces; Finite fields; Equations solvable by radicals; Transcendental extensions and transcendence degree; Algebraic number fields.

- References:**
1. D. Dummit and R. Foote, Abstract Algebra, 1991.
 2. J. Rotman, Advanced Modern Algebra, 2002.
 3. M. Artin, Algebra, 1991.
 4. T. Hungerford, Algebra, 2003.