Fall 2018

Regular graduate courses

PMATH 651: Measure and Integration (TBA) General measures, measurability, Caratheodory extension theorem and construction of measures, integration theory, convergence theorems, $L^p$ spaces, absolute continuity, differentiation of monotone functions, Radon-Nikodym theorem, product measures, Fubini's theorem, signed measures, Urysohn's lemma, Riesz Representation theorems for classical Banach spaces.

PMATH 665: Geometry of Manifolds (TBA) Point-set topology; smooth manifolds, smooth maps and tangent vectors; the tangent bundle; vector fields, tensor fields and differential forms. Other topics may include: de Rham cohomology; Frobenius Theorem; Riemannian metrics, connections and curvature.

PMATH 733: Model Theory and Set Theory (TBA) Model theory: the semantics of first order logic including the compactness theorem and its consequences, elementary embeddings and equivalence, the theory of definable sets and types, quantifier elimination, and omega-stability. Set theory: well-orderings, ordinals, cardinals, Zermelo-Fraenkel axioms, axiom of choice, informal discussion of classes and independence results.


Special topics graduate courses

PMATH 930: Topics in Logic: Computability theory (B. Csima)

PMATH 950: Topics in Analysis: Completely bounded maps (V. Paulsen)
This course will cover the principal results, motivations, and ideas in the theories of completely bounded and completely positive maps, dilation theory, operator spaces and operator systems. We will assume that students have a basic background in functional analysis along with an introduction to C*-algebras. Grades will be based on homework and a final project.

Topics to be covered include:

• positive and completely positive maps,
• Stinespring's dilation theorem, Arveson’s extension theorem, and their completely bounded generalizations,
• von Neumann’s inequality and its’ generalizations,
• Kadison’s and Halmos’ similarity problems,
• injective envelopes,
• abstract characterizations of operator systems, operator spaces, and operator algebras,
• some special operator spaces,
• Pisier’s similarity and factorization degree

PMATH 950: Topics in Analysis: Induced representations (N. Spronk)

PMATH 965: Topics in Geometry & Topology: Geometry and Dynamics of Integrable Systems (R. Moraru)

Winter 2019

Regular graduate courses

PMATH 641: Algebraic Number Theory (TBA) An introduction to algebraic number theory; unique factorization, Dedekind domains, class numbers, Dirichlet's unit theorem, solutions of Diophantine equations.

PMATH 650: Lebesgue Integration and Fourier Analysis (TBA) Lebesgue measure on the line, the Lebesgue integral, monotone and dominated convergence theorems, LP spaces, completeness and dense subspaces; separable Hilbert space, orthonormal bases; Fourier analysis on the circle, Dirichlet kernel, Riemann-Lebesgue lemma, Fejer's theorem and convergence of Fourier series.

PMATH 651: Measure and Integration (TBA) General measures, measurability, Caratheodory extension theorem and construction of measures, integration theory, convergence theorems, LP spaces, absolute continuity, differentiation of monotone functions, Radon-Nikodym theorem, product measures, Fubini's theorem, signed measures, Urysohn's lemma, Riesz Representation theorems for classical Banach spaces.

PMATH 667: Algebraic Topology (TBA) Topological spaces and topological manifolds; quotient spaces; cut and paste constructions; classification of two-dimensional manifolds; fundamental group; homology groups. Additional topics may include: covering spaces; homotopy theory; selected applications to knots and combinatorial group theory.


PMATH 863: Introduction to Lie Groups and Lie Algebras (S. New) An introduction to matrix Lie groups and their associated Lie algebras: geometry of matrix Lie groups; relations between a matrix Lie group and its Lie algebra; representation theory of matrix Lie groups.

Special topics graduate courses

PMATH 940: Topics in Number Theory: Elliptic Curves (J. Wang)

PMATH 945: Topics in Algebra: Algebraic constructions (J. Bell)

PMATH 950: Topics in Analysis: Group C*-algebras (M. Kennedy)

PMATH 965: Topics in Geometry and Topology: Special Holonomy (S. Karigiannis) This is an advanced topics course on the differential geometry of Riemannian manifolds with special structures, focusing in particular on U(m), SU(m), and G2 structures. (Course Description attached below)
Topics in Geometry and Topology:
Special Riemannian Structures

WINTER 2019

- Instructor: Spiro Karigiannis
  Email: karigiannis@uwaterloo.ca
- Telephone: 519-888-4567 ext 32810
- Office: MC 5326
- Office Hours: TBA, or by appointment
- Course Lectures: TBA, in MC TBA
- Course Website: UW-LEARN

Course description: This is an advanced topics course on the differential geometry of Riemannian manifolds with special structures, focusing in particular on $U(m)$, $SU(m)$, and $G_2$ structures.

Prerequisites: In addition to undergraduate level complex analysis, real analysis, linear algebra, and metric space topology, the following graduate level courses are required:

- PMATH 665: Geometry of Manifolds is an absolutely required prerequisite.
- A course in algebraic topology (such as PMATH 667) is helpful but not strictly necessary.

Detailed list of topics (tentative and subject to change)

Part One: Riemannian holonomy, curvature, and topology

5. Locally reducible metrics and their holonomy. Locally symmetric metrics and their holonomy.
6. Berger’s list of holonomy groups for not locally reducible, not locally symmetric Riemannian metrics.

Part Two: Manifolds with $U(m)$ or $SU(m)$ structure

10. The $\partial$ and $\bar{\partial}$ operators. Dolbeault cohomology of complex manifolds.
11. Kähler manifolds. Equivalent characterizations of the Kähler condition.
12. The Kähler identities. The Hodge decomposition of the cohomology of Kähler manifolds.

Part Three: Manifolds with $G_2$ structure

16. Algebra of the octonions $\mathbb{O}$. Cross product on $\mathbb{R}^7 \cong \text{Im}(\mathbb{O})$. Comparison with $\mathbb{R}^3 \cong \text{Im}(\mathbb{H})$ case.
17. Manifolds with $G_2$ structure. Decomposition of forms in the presence of a $G_2$-structure.
18. The torsion of a $G_2$ structure. Identities relating the curvature and torsion of $G_2$ structures.
22. Dolbeault-type cohomology for manifolds with $G_2$ structures, following Fernández–Ugarte.
23. Examples of noncompact $G_2$ manifolds. The Bryant–Salamon cohomogeneity one construction.
24. Sketch proof of Joyce’s existence theorem for torsion-free $G_2$ structures by perturbation.
26. The smoothness of the moduli space of $G_2$ manifolds via the implicit function theorem by Joyce.
27. Variational characterization of $G_2$ manifolds by Hitchin.
Textbook
There is no official textbook. I will be using many different sources for this course. Some of these are:

- Huybrechts; Complex Geometry; Springer.
- Joyce; Compact Manifolds with Special Holonomy; Oxford University Press.
- Karigiannis; Introduction to $G_2$-geometry; lecture notes.
- Morianu; Lectures on Kähler Geometry; European Mathematical Society.

Marking scheme
Course marks will be determined as follows.

- Assignments: 70% (five assignments, about one every two weeks, worth 14% each)
- Paper/presentation (approximately 10–15 pages, typewritten; worth 20% for paper and 10% for presentation)

Please note that you are encouraged to work together with your classmates on the assignment problems, but you must write up and turn in your own solutions to the problems.

Academic offenses

**Academic Integrity:** In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. Please see [http://www.uwaterloo.ca/academicintegrity/](http://www.uwaterloo.ca/academicintegrity/) for more information.

**Grievance:** A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70 - Student Petitions and Grievances, Section 4, [http://www.adm.uwaterloo.ca/infosec/Policies/policy70.htm](http://www.adm.uwaterloo.ca/infosec/Policies/policy70.htm). When in doubt please be certain to contact the departments administrative assistant who will provide further assistance.

**Discipline:** A student is expected to know what constitutes academic integrity, to avoid committing academic offenses, and to take responsibility for his/her actions. A student who is unsure whether an action constitutes an offense, or who needs help in learning how to avoid offenses (e.g., plagiarism, cheating) or about rules for group work/collaboration should seek guidance from the course professor, academic advisor, or the Undergraduate Associate Dean. For information on categories of offenses and types of penalties, students should refer to Policy 71, Student Discipline, [http://www.adm.uwaterloo.ca/infosec/Policies/policy71.htm](http://www.adm.uwaterloo.ca/infosec/Policies/policy71.htm). For typical penalties check [Guidelines for the Assessment of Penalties](http://www.adm.uwaterloo.ca/infosec/guidelines/penaltyguidelines.htm).

**Avoiding Academic Offenses:** Most students are unaware of the line between acceptable and unacceptable academic behaviour, especially when discussing assignments with classmates and using the work of other students. For information on commonly misunderstood academic offenses and how to avoid them, students should refer to the Faculty of Mathematics Cheating and Student Academic Discipline Policy, [http://www.math.uwaterloo.ca/navigation/Current/cheating_policy.shtml](http://www.math.uwaterloo.ca/navigation/Current/cheating_policy.shtml).

**Appeals:** A student may appeal the finding and/or penalty in a decision made under Policy 70 - Student Petitions and Grievances (other than regarding a petition) or Policy 71 - Student Discipline if a ground for an appeal can be established. Read Policy 72 - Student Appeals, [http://www.adm.uwaterloo.ca/infosec/Policies/policy72.htm](http://www.adm.uwaterloo.ca/infosec/Policies/policy72.htm).

Note for students with disabilities
The AccessAbility Services (AS) Office, located in Needles Hall, Room 1132, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with the AS Office at the beginning of each academic term.