uWaterloo PURE MATH Graduate Course Offerings: Fall 2019-Winter 2020
***** These offerings are fairly certain but still subject to change. *****
***** The course instructor (when known) is in parenthesis after the course title. *****

Fall 2019 Regular graduate courses

PMATH 651: Measure and Integration (V. Paulsen)
General measures, measurability, Caratheodory extension theorem and construction of measures, integration theory, convergence theorems, LP spaces, absolute continuity, differentiation of monotone functions, Radon-Nikodym theorem, product measures, Fubini's theorem, signed measures, Urysohn's lemma, Riesz Representation theorems for classical Banach spaces.

PMATH 665: Geometry of Manifolds (S. New)
Point-set topology; smooth manifolds, smooth maps and tangent vectors; the tangent bundle; vector fields, tensor fields and differential forms. Other topics may include: de Rham cohomology; Frobenius Theorem; Riemannian metrics, connections and curvature.

PMATH 740: Analytic Number Theory (M. Rubinstein)
Summation methods; analytic theory of the Riemann zeta function; Prime Number Theorem; primitive roots; quadratic reciprocity; Dirichlet characters and infinitude of primes in arithmetic progressions; assorted topics.

PMATH 745: Representations of Finite Groups (B. Madill)
Basic definitions and examples: subrepresentations and irreducible representations, tensor products of representations. Character theory. Representations as modules over the group ring, Artin-Wedderburn structure theorem for semisimple rings. Induced representations, Frobenius reciprocity, Mackey's irreducibility criterion.

PMATH 753: Functional Analysis (N. Spronk)
Banach and Hilbert spaces, bounded linear maps, Hahn-Banach theorem, open mapping theorem, closed graph theorem, topologies, nets, Hausdorff spaces, Tietze extension theorem, dual spaces, weak topologies, Tychonoff's theorem, Banach-Alaoglu theorem, reflexive spaces.

Special topics graduate courses

PMATH 930: Topics in Logic: Model Theory of Fields with Operators (R. Moosa)
A differential field is a field equipped with a linear operator satisfying the Leibniz rule, while a difference field is one equipped with a ring endomorphism. Over the last 30 years, the model theory of such structures has played a significant role in applications to algebra, geometry, and number theory. This course will be an introduction to the model theory of fields equipped with various operators, with an eye toward these applications. Some model theory (including the compactness theorem and quantifier elimination) and algebra (especially fields and polynomial rings) will be prerequisites.

PMATH 940: Topics in Number Theory: Diophantine Equations & Diophantine Inequalities (C. Stewart)
In 1934 Gelfond and Schneider independently proved that if α is algebraic and different from 0 and 1 and β is algebraic and irrational then α^{β} is transcendental. This solved Hilbert's seventh problem. In 1967 Baker proved that if $\alpha_1, \dots, \alpha_n$ are algebraic numbers and different from 0 and 1 and $\beta_1, \dots, \beta_n$ are algebraic numbers for which 1, $\beta_1, \dots, \beta_n$ are linearly independent over the rationals then $\alpha_1^{\beta_1} \dots \alpha_n^{\beta_n}$ is transcendental. It follows from Baker's proof, for which he was awarded a Fields Medal, that a number of Diophantine equations can be solved effectively. In addition a number of previously intractable Diophantine inequalities can be resolved. Applications of Baker's estimates for linear forms in the logarithms of algebraic numbers continue to be found and in this course we shall treat some of them to show the power of the ideas behind them. For instance, we shall give effective bounds for solutions of the Thue equation and discuss Tijdeman's results on the gaps between integers composed of a fixed set of primes.

PMATH 945: Topics in Algebra: Approximate representation theory of groups and nonlocal games (W. Slofstra)
A finitely-presented group is a group defined by a finite set of generators and relations. In elementary terms, a representation of a finitely-presented group is an assignment of matrices to generators such that the matrices satisfy the relations of the group. Representation theory is a central topic in mathematics, and there's a lot to be said about representations of finitely-presented groups. But what happens if we have an assignment of matrices to generators which only approximately satisfies the defining relations? For instance, is such an assignment always close to an actual representation of the group? This is a basic question of approximate representation theory, an active research area with natural applications to quantum information.

In this course, we'll cover some of the basics of approximate representation theory, as well as some recent research results, including the Gowers-Hatami stability theorem and the existence of a group which is non-appoximable in the Frobenius norm. We'll also cover some applications to quantum information, and in particular to robust self-testing in nonlocal games.

Sample topics: - Approximate representations, Ulam stability, Amenable groups and Kazhdan's stability argument, Approximation properties of groups: sofic, hyperlinear, and MF groups, Stability of relations, Gowers-Hatami theorem and stability of finite groups, Property T and k-Kazhdan groups, De Chiffre-Glebsky-Lubotzky-Thom: non-approximable group in the Frobenius norm, Hyperlinear profile, Applications to quantum information: self-testing in non-local games, robustness, and entanglement functions, Stability results from non-local games: the linearity test and the low-degree test

PMATH 945: Topics in Algebra: Category theory and Homological Algebra (J. Bell)
We cover categories, functors, natural transformations, opposite categories, adjoints and Tensor-Hom adjunction, Yonedas lemma, limits and colimits, the Govorov-Lazard theorem and filtered subcategories, abelian categories, projective modules and vector bundles, injective modules, complexes, derived functors, Ext and Tor.

Winter 2020 Regular graduate courses

PMATH 632: First order logic and computability (TBA)
The concepts of formal provability and logical consequence in first order logic are introduced, and their equivalence is proved in the soundness and completeness theorems. Goedel's incompleteness theorem is discussed; making use of the halting problem of computability theory. Relative computability and the Turing degrees are further studied.

PMATH 646: Introduction to Commutative Algebra (TBA)
Module theory: classification of finitely generated modules over PIDs, exact sequences and tensor products, algebras, localisation, chain conditions. Primary decomposition, integral extensions, Noether's normalisation lemma, and Hilbert's Nullstellensatz.

PMATH 650: Lebesgue Integration and Fourier Analysis: (TBA)
Lebesgue measure on the line, the Lebesgue integral, monotone and dominated convergence theorems, LP spaces, completeness and dense subspaces; separable Hilbert space, orthonormal bases; Fourier analysis on the circle, Dirichlet kernel, Riemann-Lebesgue lemma, Fejer's theorem and convergence of Fourier series.

PMATH 651: Measure and Integration (TBA)
General measures, measurability, Caratheodory extension theorem and construction of measures, integration theory, convergence theorems, LP spaces, absolute continuity, differentiation of monotone functions, Radon-Nikodym theorem, product measures, Fubini's theorem, signed measures, Urysohn's lemma, Riesz Representation theorems for classical Banach spaces.

PMATH 667: Algebraic Topology (TBA)
Topological spaces and topological manifolds; quotient spaces; cut and paste constructions; classification of two-dimensional manifolds; fundamental group; homology groups. Additional topics may include: covering spaces; homotopy theory; selected applications to knots and combinatorial group theory.

PMATH 764: Introduction to Algebraic Geometry (TBA)
An introduction to algebraic geometry through the theory of algebraic curves. General algebraic geometry: affine and projective algebraic sets, Hilbert's Nullstellensatz, co-ordinate rings, polynomial maps, rational functions and local rings. Algebraic curves: affine and projective plane curves, tangency and multiplicity, intersection numbers, Bezout's theorem and divisor class groups.

Special topics graduate courses

PMATH 940: Topics in Number Theory: A Second Course in Analytic Number Theory (M. Rubinstein)

PMATH 950: Topics in Analysis: Fractal Geometry (K. E. Hare)
Fractals are sets which typically have fine detail, are self-similar in some sense and have a fractal dimension. Fractals are known to abound in both nature and mathematics.

The mathematics used to study fractal sets has grown tremendously in recent years. This course will introduce students to some of these mathematical ideas and consider applications to areas such as harmonic analysis, probability theory and differential equations.

Topics may include: Construction of Hausdorff measure, Hausdorff, box and Assouad dimensions, Iterated function systems, self-similar sets and measures, Local dimensions and multi-fractal analysis, Bernoulli convolutions, Laplacians on fractals, Besicovitch and Kakeya sets.

PMATH 950: Topics in Analysis: Elements of Random Matrix Theory (A. Nica)
The study of random matrices is a vigorous area of current research which offers interesting ideas to reflect on, and interesting problems to work on for a pure mathematician. Here is an outline of a few such ideas that we would like to discuss in this course.

In order to talk about random matrices, one needs to have a space of matrices, endowed with a probability measure. For someone with background in analysis, a natural first example of this kind is found by looking at a compact matrix group (e.g. the group $\mathcal{U}(n)$ of unitary $n \times n$ matrices) endowed with its Haar measure. We will start by studying how integration is performed in this kind of framework, via a method which goes under the name of "Weingarten calculus".

Our next job will be to take on some elements of spectral theory for complex Hermitian random matrices. I will streamline this part of the course around the discussion of statistical properties for the eigenvalues of a sum of two Hermitian random matrices. To put the things into perspective: if we are given two non-random Hermitian $n \times n$ matrices A and B, a well-known linear algebra result called "Horn's conjecture" provides precise conditions on how the eigenvalues of $A + B$ are constrained via inequalities involving the eigenvalues of A and those of B. In order to "randomize" our setting, what we do (for the same A and B as above) is to consider the matrix $A + UBU^*$, where U is a random unitary matrix sampled according to the Haar measure of the group $\mathcal{U}(n)$. We will study the distribution of eigenvalues of the random matrix $A + UBU^*$, and we will find that for large values of n (or rather: when we make $n \to \infty$, in a sense which will be made precise in the course) this is governed by a certain binary operation "$\boxplus$" with probability distributions, called free additive convolution.

We will then pay some special attention to the most basic instance of a random Hermitian matrix, the so-called "Gaussian ensemble", or GUE for short. We will see how a classical analysis gadget called "Hermite polynomials" can be used in the study of an $n \times n$ GUE matrix. Also, in a context where we once again make $n \to \infty$, we will discuss the behaviour of eigenvalues that are "in the bulk" or "at the edge" of the spectrum of the GUE.

PMATH 965: Topics in Geometry and Topology: Gauge Theory (R. Moraru)