PM 950 Topics in Analysis: 
Differential Analysis 
WINTER 2017 

Tentative and Subject to Change

• Instructor: Spiro Karigiannis 
• Telephone: 519-888-4567 ext 32810 
• Office Hours: TBA 
• Course Lectures: Tuesdays/Thursdays 2:30pm–3:50pm, in MC 5479 
• Course Website: TBA 

Course description: This is an advanced topics course in analysis, focusing on concepts that are used to prove existence and uniqueness theorems for elliptic and parabolic partial differential equations on subsets of $\mathbb{R}^n$. We will discuss the Hölder ($C^{k,\alpha}$) and Sobolev ($W^{p,k}$) spaces, the embedding theorems relating these Banach spaces, their construction using Fourier transforms, distributions, and weak derivatives; and the fundamental a priori “elliptic regularity” estimates in both the Hölder and Sobolev settings. Applications to complex function theory (such as the mean value property of harmonic functions) will also be discussed. We will also prove the inverse and implicit function theorems for general Banach spaces, and use these, together with a priori estimates, to prove existence of zeroes for certain nonlinear elliptic operators using the “continuity method”. If time permits, we will also consider Harnack inequalities and parabolic evolution equations.

Prerequisites: In addition to the usual undergraduate pure mathematics preparation (including in particular: complex analysis, real analysis, linear algebra, and metric space topology) the following graduate level courses are required:

• PMATH 753: Functional Analysis is a required prerequisite.
• A course in measure theory should be at least taken concurrently. [This can be either PMATH 650: Lebesgue Integration and Fourier Analysis or PMATH 651: Measure and Integration.]

Topics to be covered include:

• Schwartz class functions; distributions; Fourier transforms; convolution; mollification; pseudo-differential operators.
• Compact operators; Fredholm operators; the Fredholm alternative.
• Sobolev spaces ($W^{p,k}$) and Hölder spaces ($C^{k,\alpha}$); Sobolev Embedding Theorem; Kondrakov–Rellich Theorem.
• Laplacians; elliptic operators; the strong and weak maximum principles.
• Harmonic function theory; mean value property; Green's functions.
• Schauder estimates; Hölder elliptic regularity; the Dirichlet problem.
• Calderon–Zygmund inequality; Sobolev elliptic regularity; a priori estimates.
• Possible additional topics: Harnack inequalities; parabolic theory.

Textbook

There is no official textbook. I will be using many different sources for this course. Some of these are:

• Evans; Partial Differential Equations; American Mathematical Society.
• Gilbarg and Trudgier; Elliptic Partial Differential Equations of Second Order; Springer-Verlag.
• Gilkey; Invariance Theory, the Heat Equation, and the Atiyah-Singer Index Theorem, freely (and legally) available here: [http://www.emis.de/monographs/gilkey/](http://www.emis.de/monographs/gilkey/)
• Melrose; Graduate Analysis; lecture notes from an M.I.T. course.
• Mitrea; Distributions, Partial Differential Equations, and Harmonic Analysis; Springer-Verlag.
• Morrey; Multiple Integrals in the Calculus of Variations; Springer-Verlag.
• Viaclokiy; PDE; lecture notes from an M.I.T. course.
Marking scheme

Course marks will be determined by one of the following two schemes. [I will decide which scheme I will use by the end of the fourth week of lectures, depending on how many students are taking this course for credit.]

- Possibility 1: Assignments: 100% (six assignments, one every two weeks, worth 16.67% each)
- Possibility 2: Assignments: 70% (five assignments, about one every two weeks, worth 14% each), and one project and presentation (approximately 10–15 pages, typewritten; worth 20% for the paper and 10% for the presentation)

Please note that you are encouraged to work together with your classmates on the assignment problems, but you must write up and turn in your own solutions to the problems.

Academic offenses

Academic Integrity: In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. Please see http://www.uwaterloo.ca/academicintegrity/ for more information.

Grievance: A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70 - Student Petitions and Grievances, Section 4. When in doubt please be certain to contact the departments administrative assistant who will provide further assistance.

Discipline: A student is expected to know what constitutes academic integrity, to avoid committing academic offenses, and to take responsibility for his/her actions. A student who is unsure whether an action constitutes an offense, or who needs help in learning how to avoid offenses (e.g., plagiarism, cheating) or about rules for group work/collaboration should seek guidance from the course professor, academic advisor, or the Undergraduate Associate Dean. For information on categories of offenses and types of penalties, students should refer to Policy 71, Student Discipline. For typical penalties check Guidelines for the Assessment of Penalties.

Avoiding Academic Offenses: Most students are unaware of the line between acceptable and unacceptable academic behaviour, especially when discussing assignments with classmates and using the work of other students. For information on commonly misunderstood academic offenses and how to avoid them, students should refer to the Faculty of Mathematics Cheating and Student Academic Discipline Policy.

Appeals: A student may appeal the finding and/or penalty in a decision made under Policy 70 - Student Petitions and Grievances (other than regarding a petition) or Policy 71 - Student Discipline if a ground for an appeal can be established. Read Policy 72 - Student Appeals.

Note for students with disabilities

The AccessAbility Services (AS) Office, located in Needles Hall, Room 1132, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with the AS Office at the beginning of each academic term.