Course Information
PMath 651 (Measure and Integration), Fall Term 2016

Room and time: M W 1–2:20 pm, in QNC 1506

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Course description. This course provides a basic introduction to Lebesgue’s theory of measure and integration. We will be primarily concerned with a framework where (instead of merely considering the Lebesgue measure on \( \mathbb{R} \)) we look at a positive measure \( \mu \) on a metric space \( X \). We will spend a bit of time to understand how such measures \( \mu \) can be constructed, and why using this more general framework is useful.

We will then do a concise overview of Lebesgue’s method for integrating measurable functions, upgraded to the \((X, \mu)\) framework. The main ideas for constructing the integral are pretty much the same as in the special case of Lebesgue integration on \( \mathbb{R} \), and same is true about the statements and proofs of the very important convergence theorems of Lebesgue (the “monotone” and the “dominated” convergence theorem).

The remaining part of the course will be devoted to the following three topics.

- Absolute continuity, the Radon-Nikodym theorem and the Lebesgue decomposition theorem. Here we will also discuss some basic applications of Radon-Nikodym (for instance to duality for \( L^p \)-spaces).
- Product measures and the theorem of Fubini concerning iterated integrals.
- Radon integrals and the Riesz representation theorem. Here we will focus for simplicity on the case when \( X \) is a compact metric space. It is very useful that finite positive measures on \( X \) can be defined by prescribing how they integrate continuous functions. We will examine how this goes, in the form of a theorem of Riesz which identifies the dual of \( C(X) \) as a space of signed measures on \( X \).

Textbook. There is no required textbook. I will attempt to make the sequence of lectures clear and well-organized, so that your notes themselves can be used as primary reference when you study this material. On occasion we will study excerpts of some notes prepared by Professor Forrest for a preceding edition of the course (these excerpts will be posted on the Learn web-site of the course).

If you are looking for additional sources, some “classic” textbooks which cover the material of this course are for instance the one by H.L. Royden (Real Analysis, 3rd edition) and the one by W. Rudin (Real and Complex Analysis, 3rd edition).

Course grade. The weights are: 20% homework assignments; 20% midterm exam; 50% final exam; 10% for a project and presentation on a topic related to the course (to be picked in agreement with the instructor).
It was mandated by the University Senate that every course outline must contain the following text.

_Academic Integrity:_ In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. Check [http://www.uwaterloo.ca/academicintegrity/](http://www.uwaterloo.ca/academicintegrity/) for more information.

_Grievance:_ A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70, Student Petitions and Grievances, Section 4, [http://www.adm.uwaterloo.ca/infosec/Policies/policy70.htm](http://www.adm.uwaterloo.ca/infosec/Policies/policy70.htm). When in doubt please be certain to contact the department’s administrative assistant who will provide further assistance.

_Discipline:_ A student is expected to know what constitutes academic integrity to avoid committing academic offenses and to take responsibility for his/her actions. A student who is unsure whether an action constitutes an offense, or who needs help in learning how to avoid offenses (e.g., plagiarism, cheating) or about ”rules” for group work/collaboration should seek guidance from the course professor, academic advisor, or the undergraduate associate dean. For information on categories of offenses and types of penalties, students should refer to Policy 71, Student Discipline, [http://www.adm.uwaterloo.ca/infosec/Policies/policy71.htm](http://www.adm.uwaterloo.ca/infosec/Policies/policy71.htm). For typical penalties check Guidelines for the Assessment of Penalties, [http://www.adm.uwaterloo.ca/infosec/guidelines/penaltyguidelines.htm](http://www.adm.uwaterloo.ca/infosec/guidelines/penaltyguidelines.htm).

_Apppeals:_ A decision made or penalty imposed under Policy 70, Student Petitions and Grievances (other than a petition) or Policy 71, Student Discipline may be appealed if there is a ground. A student who believes he/she has a ground for an appeal should refer to Policy 72, Student Appeals, [http://www.adm.uwaterloo.ca/infosec/Policies/policy72.htm](http://www.adm.uwaterloo.ca/infosec/Policies/policy72.htm).

_Note for students with disabilities:_ The Office for Persons with Disabilities (OPD), located in Needles Hall, Room 1132, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with the OPD at the beginning of each academic term.