Topics in Analysis:
Introduction to stochastic integration

Instructor: Alexandru Nica
Office: MC 5334  Office hour: Tuesday, 4-5:30 pm.
Email: anica@uwaterloo.ca.

Room and time: M W F 11:30 am –12:20 pm, in M3-2134.

Course Description.
Stochastic integrals give a generalization of the classical Riemann-Stieltjes integral
"\( \int_{a}^{b} f(t) \, dg(t) \)", where both \( f \) and \( g \) become random functions. The goal of the course
is to provide a basic introduction to such integrals, which is rigorous and is, at the same
time, as clear and simple as possible. We will start by discussing the mathematical tools
that are necessary for the job: Gaussian random vectors, the Brownian motion, and the
concept of martingale. Once this is done, we will begin to construct stochastic integrals:
first the Wiener integral, then various versions (increasing in generality) of the Itô integral;
interestingly enough, all these integrals can be thought of, at some conceptual level, as
isometric linear maps between suitable \( L^2 \)-spaces. Once we reach a satisfactory version of
stochastic integral, we will prove for it the celebrated Itô formula which is, in some sense,
the counterpart in stochastic calculus for the chain rule of classical calculus. In the final
part of the course (depending on how much time is left available) we will look at some
applications of Itô’s formula.

Recommended textbooks.
- “Introduction to stochastic integration” by Hui-Hsiung Kuo, Springer (Universitext
  Series), 2006.
- (If you read French.) “Mouvement brownien, martingales et calcul stochastique” by

These are two nice little books, which also have the advantage of being available online
at the university library. In the beginning of the course I will sample introductory items
out of both these books (I will do that in such a way that the lectures are self-contained).
From the moment when we arrive at the Itô integral I will follow mostly the book by Kuo,
with the aim to cover at least the material in its Chapters 4,5 and 7.

Prerequisites. The required prerequisite is a solid course in abstract measure and
integration theory (PMath 451/651, or the equivalent course from another university). Some
prior exposure to Gaussian random variables and to the Brownian motion will be useful,
but is not a prerequisite for taking the course.

The course grade will be based on participation in class and homework assignments
(65%) and some form of written final examination (35%).
It was mandated by the University Senate that every course outline must contain the following text.

**Academic Integrity:** In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. For more information, check http://www.uwaterloo.ca/academicintegrity/

**Grievance:** A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70, Student Petitions and Grievances, Section 4, http://www.adm.uwaterloo.ca/infosec/Policies/policy70.htm. When in doubt please be certain to contact the department’s administrative assistant who will provide further assistance.

**Discipline:** A student is expected to know what constitutes academic integrity to avoid committing academic offenses and to take responsibility for his/her actions. A student who is unsure whether an action constitutes an offense, or who needs help in learning how to avoid offenses (e.g., plagiarism, cheating) or about "rules" for group work/collaboration should seek guidance from the course professor, academic advisor, or the undergraduate associate dean. For information on categories of offenses and types of penalties, students should refer to Policy 71, Student Discipline, http://www.adm.uwaterloo.ca/infosec/Policies/policy71.htm. For typical penalties check Guidelines for the Assessment of Penalties, http://www.adm.uwaterloo.ca/infosec/guidelines/penaltyguidelines.htm.

**Appeals:** A decision made or penalty imposed under Policy 70, Student Petitions and Grievances (other than a petition) or Policy 71, Student Discipline may be appealed if there is a ground. A student who believes he/she has a ground for an appeal should refer to Policy 72, Student Appeals, http://www.adm.uwaterloo.ca/infosec/Policies/policy72.htm.

**Note for students with disabilities:** The Office for Persons with Disabilities (OPD), located in Needles Hall, Room 1132, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with the OPD at the beginning of each academic term.