

# Pure Math 445/745

## Representations of Finite Groups

**Class times and place:** TTh from 11:30–12:50AM in MC 4064

**Instructor:** Ben Webster

**Office:** MC 5429

**Office Hours:** Tuesday 1–2PM, Thursday 10–11AM

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### Description

The course introduces students to the representation theory of finite groups. It begins with a discussion of the elementary properties of linear representations of finite groups, including subrepresentations and irreducibility. In particular, we discuss the regular representation and its connection to the group algebra. Next comes character theory, including the statement and proof of Schur's Lemma, the orthogonality relations, and the canonical decomposition of a representation. With the notions of induced representations, the course now has the tools to explore a variety of examples. Finally, we'll cover applications to the structure of groups, and briefly discuss how to apply representation theory to molecular vibration.

### Texts

The primary text for the class will be “Representations and Characters of Groups” by Gordon James and Martin Liebeck. I suggest having access to a physical or electronic copy of this book. You can refer to other online notes such as those of [Teleman](#) and [Webb](#).

Announcements and additional information will be posted to LEARN and emailed to students; students are responsible for checking these.

### Grades and assignments

The grade for the class will be primarily based on:

1. **Weekly homework (40 %):** This will be due every week when practical (so for example, not the week of Thanksgiving). This will be administered through Crowdmark, so students can submit typed work or images of handwritten work.
2. **A take-home mini-midterm exam (15 %):** This will be in mid-October; it will be administered like HW through Crowdmark, and will be about half the length of the final. It will be open note and open text, but you should consult the instructor about using other references.
3. **The final exam (40 %):** This will be administered as normal in person during the finals period. It will be cumulative (covering the whole material of the class).
4. **Participation (5 %):** I'll ask students to work on problems in small groups in class (1%), to send me at least two questions via email over the course of the semester (2%), and I will take attendance at all class meetings (2%).

### Lecture schedule

Sept. 6	Intro to the course	
Sept. 11	Group theory and linear algebra background	J&L, Chapters 1–2
Sept. 13	Representations and $FG$ -modules	J&L, Chapters 3–4

Sept. 18	Submodules and (ir)reducibility	J&L, Chapters 5–6
Sept. 20	$FG$ -homomorphisms and Maschke’s theorem	J&L, Chapters 7–8
Sept. 25	Schur’s Lemma	J&L, Chapter 9
Sept. 27	The regular representation	J&L, Chapters 10–11
Oct. 2	Conjugacy classes and characters	J&L, Chapters 12–13
Oct. 4	Inner products and orthogonality	J&L, Chapters 14
Oct. 11	Character tables	J&L, Chapters 15–16
Oct. 16	Examples of character tables	J&L, Chapters 17–18
Oct. 18	Tensor product and restriction to a subgroup	J&L, Chapters 19–20
Oct. 23	Induction of representations and characters	J&L, Chapter 21
Oct. 25	Permutation representations	J&L, Chapter 29
Oct. 30	Integrality	J&L, Chapter 22
Nov. 1	Real representations	J&L, Chapter 23
Nov. 6	Schur indicator	Conrad
Nov. 8	The Artin-Wedderburn theorem	Webb, Chapter 2
Nov. 13	Further examples of character tables	J&L, Chapter 24–26
Nov. 15	Character table of some simple groups	J&L, Chapter 27–28
Nov. 20	Applications to group theory	J&L, Chapter 30–31
Nov. 22	Molecular vibration	J&L, Chapter 32

## Know your Rights and Responsibilities

- Academic Integrity: In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. [Check [www.uwaterloo.ca/academicintegrity/](http://www.uwaterloo.ca/academicintegrity/) for more information.]
- Grievance: A student who believes that a decision affecting some aspect of his or her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70, Student Petitions and Grievances, Section 4, [www.adm.uwaterloo.ca/infosec/Policies/policy70.htm](http://www.adm.uwaterloo.ca/infosec/Policies/policy70.htm). When in doubt please be certain to contact the department’s administrative assistant who will provide further assistance.
- Discipline: A student is expected to know what constitutes academic integrity [check [www.uwaterloo.ca/academicintegrity/](http://www.uwaterloo.ca/academicintegrity/)] to avoid committing an academic offence, and to take responsibility for his/her actions. A student who is unsure whether an action constitutes an offence, or who needs help in learning how to avoid offences (e.g., plagiarism, cheating) or about “rules” for group work/collaboration should seek guidance from the course instructor, academic advisor, or the undergraduate Associate Dean. For information on categories of offences and types of penalties, students should refer to Policy 71, Student Discipline, [www.adm.uwaterloo.ca/infosec/Policies/policy71.htm](http://www.adm.uwaterloo.ca/infosec/Policies/policy71.htm). For typical penalties check Guidelines for the Assessment of Penalties, <http://www.adm.uwaterloo.ca/infosec/guidelines/penaltyguidelines.htm>.
- Appeals: A decision made or penalty imposed under Policy 70 (Student Petitions and Grievances) (other than a petition) or Policy 71 (Student Discipline) may be appealed if there is a ground. A student who believes he/she has a ground for an appeal should refer to Policy 72 (Student Appeals), [www.adm.uwaterloo.ca/infosec/Policies/policy72.htm](http://www.adm.uwaterloo.ca/infosec/Policies/policy72.htm).
- Note for Students with Disabilities: AccessAbility Services, located in Needles Hall, Room 1132, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with AccessAbility Services at the beginning of each academic term.