

# PMATH 451/651 - Fall 2018

## Measure Theory

- INSTRUCTORS:** Matthew Kennedy  
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MC 5431
- DESCRIPTION:** General measures, measurability, Caratheodory extension theorem and construction of measures, integration theory, convergence theorems, Lebesgue spaces, absolute continuity, differentiation of monotone functions, Radon-Nikodym theorem, product measures, Fubini's theorem, signed measures, Urysohn's lemma, Riesz-Markov-Kakutani representation theorem
- PREREQUISITE:** PMATH 450/650 or equivalent
- REFERENCES:** *Real Analysis: Modern Techniques and Their Applications*  
by Gerald B. Folland  
*Measure Theory*  
by Paul Halmos  
*Real and Complex Analysis*  
by Walter Rudin
- WEBPAGE:** <https://learn.uwaterloo.ca/d21/home/403743>
- GRADING:**
- |             | <b>PMATH 451</b> |  | <b>PMATH 651</b> |
|-------------|------------------|--|------------------|
| Assignments | 40%              |  | Assignments 40%  |
| Final Exam  | 60%              |  | Talk 10%         |
|             |                  |  | Final Exam 50%   |
- ASSIGNMENTS:** Assignments will be posted approximately biweekly.
- FINAL EXAM:** There will be a final exam scheduled by the university during the fall examination period.
- NOTES:** For any missed examination, you must have a valid reason and appropriate supporting documentation. To be considered for an incomplete (INC) grade, you must have done well on all term work.

**TENTATIVE SYLLABUS:**

<b>TOPIC</b>	<b>DETAILS</b>
Measures	$\sigma$ -algebras, measures, outer measures and Caratheodory's theorem, Borel measures on the real line, Lebesgue-Steiltjes measure
Integration	Measurable functions, integrals of non-negative functions, monotone convergence theorem, Fatou's lemma, integration of complex-valued functions, dominated convergence theorem, modes of convergence, Egoroff's theorem, product measures, Tonelli's theorem, Fubini's theorem, higher dimensional Lebesgue integrals
Differentiation	Signed measures, Hahn and Jordan decompositions, total variation, Lebesgue-Radon-Nikodym theorem and Lebesgue decomposition, complex measures, differentiation, maximal functions, Lebesgue's differentiation theorem, functions of bounded variation, absolute continuity
Lebesgue Spaces	Basic theory, Holder and Minkowski inequalities, dual spaces, inequalities, containment relations
Radon Measures	Linear functionals on spaces of continuous functions, Riesz-Markov-Kakutani representation theorem, regularity and approximation, Lusin's theorem

**ACADEMIC INTEGRITY:** In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. See <http://www.uwaterloo.ca/academicintegrity/> for more information.

**GRIEVANCE:** A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. See Policy 70, Student Petitions and Grievances, Section 4 at <http://www.adm.uwaterloo.ca/infosec/Policies/policy70.htm>. When in doubt, please be certain to contact the department's administrative assistant, who will provide further assistance.

**DISCIPLINE:** A student is expected to know what constitutes academic integrity in order to avoid committing academic offenses, and to take responsibility for his/her actions. A student who is unsure whether an action constitutes an offense, or who needs help in learning how to avoid offenses (e.g., plagiarism, cheating) or about "rules" for group work/collaboration should seek guidance from the course professor, an academic advisor or the undergraduate associate dean. For information on categories of offenses and types of penalties, students should refer to Policy 71, Student Discipline at <http://www.adm.uwaterloo.ca/infosec/Policies/policy71.htm>.

**APPEALS:** A decision made or a penalty imposed under Policy 70, Student Petitions and Grievances (other than a petition) or Policy 71, Student Discipline, may be appealed if there is a ground. A student who believes he/she has a ground for an appeal should refer to Policy 72, Student Appeals at <http://www.adm.uwaterloo.ca/infosec/Policies/policy72.htm>.

**STUDENTS WITH DISABILITIES:** The AccessAbility Office, located in Needles Hall, Room 1132, collaborates with academic departments to arrange appropriate accommodations for students with disabilities, without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with the AccessAbility Office at the beginning of each academic term.