Pure Mathematics Fields and Galois Qualifying Examination
University of Waterloo
September 28, 2023

Instructions

1. Print your name and UWaterloo ID number at the top of this page, and on no other page.

2. Check for questions on both sides of each page.

3. Answer the questions in the spaces provided. If you require additional space to answer a question, please use one of the overflow pages, and refer the grader to the overflow page from the original page by giving its page number.

4. Do not write on the Crowdmark QR code at the top of each page.

5. Use a dark pencil or pen for your work.

6. All questions are equally weighted.

1. Find the degree of the splitting field over $\mathbb{Q}$ for $f(x) = x^{17} - 1$.

Extra page for answers. Please specify the question number here and the use of this page on the question page.

2. Let K be a field of 8 elements. Compute the Galois group of the Galois cover of K over the prime field of K.

Extra page for answers. Please specify the question number here and the use of this page on the question page.

3. (a) Compute the degree of the extension $K = \mathbb{Q}(\sqrt{5}, \sqrt{7})$ over $\mathbb{Q}$.

(b) Find an element $\theta \in K$ such that $K = \mathbb{Q}(\theta)$.

Extra page for answers. Please specify the question number here and the use of this page on the question page.

4. Let $f(x)$ be irreducible over $\mathbb{Q}$, and let F be its splitting field over $\mathbb{Q}$. Show that if the Galois group of F over $\mathbb{Q}$ is abelian, then $F = \mathbb{Q}(u)$ for all roots u of $f(x)$.

Extra page for answers. Please specify the question number here and the use of this page on the question page.

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