Pure Mathematics Groups and Rings Qualifying Examination
University of Waterloo
September 26, 2023

Instructions

1. Print your name and UWaterloo ID number at the top of this page, and on no other page.
2. Check for questions on both sides of each page.
3. Answer the questions in the spaces provided. If you require additional space to answer a question, please use one of the overflow pages, and refer the grader to the overflow page from the original page by giving its page number.
4. Do not write on the Crowdmark QR code at the top of each page.
5. Use a dark pencil or pen for your work.
6. All questions are equally weighted.

1. Let G be a group of the order 150. Prove that G is not simple.

Extra page for answers. Please specify the question number here and the use of this page on the question page.

2. The alternating group A_n is the group of even permutations. Prove that every finite group is a subgroup of A_n for some integer n.

Extra page for answers. Please specify the question number here and the use of this page on the question page.

3. Let R be a ring and $\phi : R \to R$ a surjective ring homomorphism. Let ϕ^m denote the composition of ϕ with itself m times. Suppose that for some m,

$$\ker(\phi^{m+1}) \subseteq \ker(\phi^m).$$

Show that ϕ is injective.

Extra page for answers. Please specify the question number here and the use of this page on the question page.

4. Let R be a commutative ring with a unit. Recall that the Jacobson radical of R, denoted $J(R)$, is the intersection of all the maximal ideals of R. Show that $x \in J(R)$ if and only if $xy - 1$ is a unit for all $y \in R$.

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