

Instructions

- 1. Print your name and UWaterloo ID number at the top of this page, and on no other page.
- 2. Check for questions on both sides of each page.
- 3. Answer the questions in the spaces provided. If you require additional space to answer a question, please use one of the overflow pages, and refer the grader to the overflow page from the original page by giving its page number.
- 4. Do not write on the Crowdmark QR code at the top of each page.
- 5. Use a dark pencil or pen for your work.
- 6. All questions are equally weighted.

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1. Find the degree of the splitting fields (as extensions of $\mathbb{Q}$) for the following irreducible rational polynomials. You do not need to justify your answer.

a) $f(x) = x^3 - 2$

b) $f(x) = x^{11} - 1$

Extra page for answers. Please specify the question number here and the use of this page on the question page.

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2. Show that if $f(x) \in \mathbb{Q}[x]$ is irreducible and the Galois group of f has size 8, then the degree of f is either 4 or 8.

Extra page for answers. Please specify the question number here and the use of this page on the question page.

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3. Let $f(x) \in \mathbb{Q}[x]$ be an irreducible polynomial of degree 3.
- a) Show that the Galois group of $f(x)$ is either S_3 or cyclic of order 3.
 - b) Show that if the Galois group of $f(x)$ is cyclic of order 3 then $f(x)$ has three real roots.

Extra page for answers. Please specify the question number here and the use of this page on the question page.

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4. Let $f(x) \in \mathbb{Q}[x]$ be an irreducible polynomial of degree 4 and let $K \subseteq \mathbb{C}$ denote the splitting field of f over $\mathbb{Q}$. Suppose that Galois group of $K/\mathbb{Q}$ is cyclic of order 4. Show that if L is a subfield of K with $[L : \mathbb{Q}] = [K : L] = 2$ then $L \subseteq \mathbb{R}$.

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