

Instructions

1. Print your name and UWaterloo ID number at the top of this page, and on no other page.
2. Check for questions on both sides of each page.
3. Answer the questions in the spaces provided. If you require additional space to answer a question, please use one of the overflow pages, and refer the grader to the overflow page from the original page by giving its page number.
4. Do not write on the Crowdmark QR code at the top of each page.
5. Use a dark pencil or pen for your work.
6. All questions are equally weighted.

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1. Let G be a group of order $231 = 3 \cdot 7 \cdot 11$.
 - a) Show that G has a unique subgroup of size 11.
 - b) Show that this subgroup of size 11 is contained in the centre of G .

Extra page for answers. Please specify the question number here and the use of this page on the question page.

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2. Let G be a group and suppose that G has a subgroup of index m . Show that G has a proper normal subgroup of index at most $m!$.

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3. Suppose that R is a commutative ring with unit. Show that if m is a maximal ideal then m is prime.

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4. Recall that the Krull dimension of a ring R is the supremum of the lengths of all chains of prime ideals, where the length of a chain

$$p_0 \subsetneq p_1 \subsetneq \cdots \subsetneq p_n$$

is n . Let R be a unique factorization domain.

- (a) Show that if p is an irreducible element of R , then the principal ideal (p) is a prime ideal.
- (b) Suppose that, in addition, R has Krull dimension 1. Show that every prime ideal is principal.

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