

Instructions

1. Print your name and UWaterloo ID number at the top of this page, and on no other page.
2. Check for questions on both sides of each page.
3. Answer the questions in the spaces provided. If you require additional space to answer a question, please use one of the overflow pages, and refer the grader to the overflow page from the original page by giving its page number.
4. Do not write on the Crowdmark QR code at the top of each page.
5. Use a dark pencil or pen for your work.
6. All questions are equally weighted.

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1. Let $f(x) = x^6 + 9x^4 - 4x^3 + 27x^2 + 36x + 31 \in \mathbb{Q}[x]$. Given that $\sqrt[3]{2} + \sqrt{-3}$ is a root of $f(x)$, prove that $f(x)$ is irreducible.

Extra page for answers. Please specify the question number here and the use of this page on the question page.

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2. Let ζ be a primitive 7^{th} root of unity. Describe every subfield of $\mathbb{Q}(\zeta)$ as $\mathbb{Q}(\alpha)$ for a *single* element $\alpha \in \mathbb{Q}(\zeta)$. *Hint*: First find a generator for $(\mathbb{Z}/7\mathbb{Z})^\times$.

Extra page for answers. Please specify the question number here and the use of this page on the question page.

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3. Let F be a finite field, let K be another field, and let $\phi, \psi: F \rightarrow K$ be field homomorphisms. Prove that $\text{Im}(\phi) = \text{Im}(\psi)$.

Extra page for answers. Please specify the question number here and the use of this page on the question page.

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4. Given a field extension $K \supseteq F$, prove that the following are equivalent:
- (a) $K \supseteq F$ is algebraic.
 - (b) For every field L with $K \supseteq L \supseteq F$, every field homomorphism from L to L which fixes F pointwise is an automorphism.

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