

Instructions

- 1. Print your name and UWaterloo ID number at the top of this page, and on no other page.
- 2. Check for questions on both sides of each page.
- 3. Answer the questions in the spaces provided. If you require additional space to answer a question, please use one of the overflow pages, and refer the grader to the overflow page from the original page by giving its page number.
- 4. Do not write on the Crowdmark QR code at the top of each page.
- 5. Use a dark pencil or pen for your work.
- 6. All questions are equally weighted.

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1. Let p and q be prime numbers (possibly equal). Let G be a group with $|G| = pq$. Suppose $a, b \in G$ are such that $H := \langle a, b \rangle$ is not cyclic. What is H ? Prove your claim.

Extra page for answers. Please specify the question number here and the use of this page on the question page.

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2. Let the group G act transitively on the set X . For each $x \in X$, write $G_x \leq G$ for the set $\{g \in G : gx = x\}$.

Prove that $G_x \leq G$. Also, prove that $G_x \trianglelefteq G$ iff we have $G_x = G_y$ for every $x, y \in X$.

Extra page for answers. Please specify the question number here and the use of this page on the question page.

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3. Given a field k , let $R(k) \subset M_2(k)$ be the subring of matrices of the form $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$. Prove that $R(\mathbb{F}_7)$ is a field and $R(\mathbb{F}_{13})$ is not a field.

Extra page for answers. Please specify the question number here and the use of this page on the question page.

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4. Prove that $(2, 1 + \sqrt{-13})$ is a prime ideal of $\mathbb{Z}[\sqrt{-13}]$ but it is not principal.

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