

Instructions

1. Print your name and UWaterloo ID number at the top of this page, and on no other page.
2. Check for questions on both sides of each page.
3. Answer the questions in the spaces provided. If you require additional space to answer a question, please use one of the overflow pages, and refer the grader to the overflow page from the original page by giving its page number.
4. Do not write on the Crowdmark QR code at the top of each page.
5. Use a dark pencil or pen for your work.
6. All questions are equally weighted.

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1. Give an example of two 4×4 nilpotent matrices that have the same minimal polynomial but are not similar.

Extra page for answers. Please specify the question number here and the use of this page on the question page.

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2. Given $A \in M_n(\mathbb{C})$ consider the linear functional on $M_n(\mathbb{C})$ given by $f_A(B) = \text{Tr}(AB)$. Prove that every linear functional on $M_n(\mathbb{C})$ is of the form f_A .

Extra page for answers. Please specify the question number here and the use of this page on the question page.

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3. Consider the vector space V of continuous functions on the interval $[-1, 1]$ and equip it with the inner product $\langle f, g \rangle = \int_{-1}^1 f(t)g(t)dt$. What is the orthogonal complement of the subspace of odd functions?

Extra page for answers. Please specify the question number here and the use of this page on the question page.

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4. Let f be a linear operator on a finite dimensional space V . Recall a vector v is cyclic for f if the vectors $\{f^n v \mid n \geq 0\}$ span V . Prove that f has a cyclic vector if and only if every linear operator g that commutes with f is a polynomial in f .

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