PMath 810 **Course Outline**
Operator Algebras **January 2025**

Instructor: L.W. Marcoux MC 5014

Chapter One: **A review of the main theorems from Banach spaces**

Chapter Two: **Banach algebras**
1. Basic theory
2. The functional calculus
3. Relative spectra

Chapter Three: **Operator algebras**
1. The algebra $\mathcal{B}(\mathfrak{X})$, $\mathfrak{X}$ a Banach space
2. The algebra $\mathcal{B}(\mathcal{H})$, $\mathcal{H}$ a Hilbert space
3. Compact operators

Chapter Four: **Commutative Banach algebras**
1. The Gelfand transform
2. Examples
3. The Jacobson radical

Chapter Five: **C*-algebras**
1. Definitions and Basic Theory
2. Elements of C^*-algebras
3. Ideals in C^*-algebras
4. The GNS Construction

Chapter Six: **von Neumann algebras**
1. An introduction
2. The Spectral Theorem for normal operators

Grading. To be determined

There is no specific text for the course, although I shall be making a typed version my notes available to you online. A preliminary version of these notes is currently available on my UW website: https://www.math.uwaterloo.ca/∼lwmarcou/.

The prerequisites for this course are: PM351 (Real Analysis I - metric space theory); PM352 (Complex Analysis); PM450 (Lebesgue measure and Fourier Analysis); PM453 (Functional Analysis). At least one course on (non-commutative) rings is also very useful. The student taking this course would be well-served to review their notes for Advanced Linear algebra.

Some other textbooks which may be helpful for PMath 810 are:

Murphy, G.J., *C*-algebras and Operator Theory*, Academic Press, 1990.

Douglas, R., *Banach algebra techniques in operator theory*, Academic Press, 1972.

Kadison, R. and Ringrose, J., *Fundamentals of the theory of operator algebras*, Academic Press, 1983.

Rudin, W., *Functional Analysis*, McGraw-Hill, 1977.

Takesaki, M., *Theory of operator algebras*, Springer Verlag, 1979.