PMATH 868 -- Connections and Riemannian Geometry
Course description – Winter 2025

---

*Instructor:* Ruxandra Moraru
*Email:* moraru@uwaterloo.ca

---

*Overview:* Review of smooth manifolds. Vector bundles. Connections and curvature, holonomy, characteristic classes. Connections on tangent bundle: torsion, geodesics, exponential map. Riemannian geometry: Levi-Civita connection, Riemannian geodesics, Hopf-Rinow Theorem. Additional topics if time permits.

*Outline of topics:* This course deals with vector bundles on smooth manifolds as well as objects on them such as connections and metrics. In Riemannian geometry, the main object of interest is the tangent bundle together with a Riemannian metric. We present the general theory of vector bundles further specializing to standard material from Riemannian geometry when appropriate. Here is a detailed outline of topics:

- *General bundle theory:* Vector bundles (definitions and basic constructions); connections, curvature, and gauge groups; covariant derivatives and holonomy; characteristic classes and Chern-Weil Theory; flat connections and representations of the fundamental group; metric connections on vector bundles; (time permitting) some important equations of gauge theory: Yang-Mills, anti-self-dual, Hermitian-Einstein.
- *Some Riemannian geometry:* Connections on the tangent bundle: torsion, geodesics, the exponential map; Riemannian metrics, the Levi-Civita connection, the Hopf-Rinow theorem.

*Prerequisites:* PMATH 465/665 (Geometry of Manifolds) or an equivalent course.

---

*Suggested texts (available online through the University of Waterloo library):*

1. L. W. Tu, *An Introduction to Manifolds*, Universitext, Springer, 2010
2. J. M. Lee, *Introduction to Smooth Manifolds*, Graduate Texts in Mathematics: 218, Springer, 2012
3. L. W. Tu, *Differential Geometry*. *Connections, Curvature, and Characteristic Classes*, Graduate Texts in Mathematics: 275, Springer, 2017
4. J. M. Lee, *Introduction to Riemannian manifolds*, Graduate Texts in Mathematics: 176, Springer, 2nd ed., 2018

*Additional references:*

1. S. Kobayashi, *Differential Geometry of Complex Vector Bundles*, Princeton, 1987
2. D. Huybrechts, *Complex Geometry: An Introduction*, Universitext, Springer, 2004
3. P. Griffiths and J. Harris, *Principles of Algebraic Geometry*, Wiley Interscience, 1978
4. J. Milnor and J. Stasheff, *Characteristic classes*, Princeton, 1974
5. M. Nakahara, *Geometry, Topology and Physics*, Boca Raton, FL : Taylor & Francis Group, 2nd ed., 2003

---

*Method of evaluation:* Your final grade will be based on 4 assignments grades for the term.