Periodica Mathematica Hungarica Vol. 42 (1–2), (2001), pp. 1–16

ANDRÁS SÁRKÖZY—A RETROSPECTIVE ON THE OCCASION OF HIS SIXTIETH BIRTHDAY

C. L. Stewart (Waterloo)

(The following is the text of a lecture given July 3, 2000, in Debrecen at the Colloquium on Number Theory in honor of the sixtieth birthday of Professors Kálmán Győry and András Sárközy.)

It is both a pleasure and a privilege to give a lecture on the mathematics of András Sárközy on this occasion. András has written over 160 papers and 4 books. He has many coauthors and has written more joint papers with Paul Erdős than any other mathematician.

I first met András twenty years ago at the Number Theory Conference of the János Bolyai Mathematical Society in Budapest in the summer of 1981. We have been friends ever since and have had great fun doing mathematics together over the years. In this talk I will survey some of András' work. Clearly I can only pick out some highlights and so I have focussed on the results that I have found most appealing.

András has worked mainly in combinatorial and analytic number theory. However his first paper "On lattice cubes in three dimensional space" which appeared in *Mat. Lapok* in 1961 was in geometry and another one of his early papers dealt with classical analysis. The problem was to find, for each positive degree n, the polynomials f for which the difference of the maximum absolute value of f and the nearest zero on $[-1,1]$ is minimal. The solution is given by those polynomials which are multiples of T_n, the n-th Chebyshev polynomial of the first kind. Lázár had claimed a proof of this result but his proof was lost with him when he died in the Second World War. Turán popularized the problem and Á. Elbert and A. Sárközy solved it [14] as fourth-year university students.

Let $a_1 < a_2 < \dots$ be a sequence of positive integers with the property that a_i does not divide a_j whenever i is different from j. In 1935 Behrend [11] proved that for such a sequence

$$\sum_{a_i \le N} a_i^{-1} << \frac{\log N}{(\log\log N)^{1/2}};$$

here $<<$ is Vinogradov's notation and signifies that the left-hand side of the symbol is less than a positive constant times the right-hand side.

Research supported in part by Grant A3528 of the Natural Sciences and Engineering Research Council of Canada.

0031-5303/01/$5.00

Akadémiai Kiadó, Budapest
Kluwer Academic Publishers, Dordrecht

Erdős, Sárközy and Szemerédi wrote a sequence of 11 papers from 1966 to 1970 [18, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33] dealing with extensions and generalizations of Behrend's result. Sárközy took up this theme later with Pomerance [54] and most recently with Ahlswede and Khachatrian [2, 3].

Another topic on which András made significant progress is that of irregularities of distribution in arithmetical progressions [10, 57, 58, 59, 63, 66]. He gave one-sided estimates improving on the work of Roth, applied his estimates to give lower bounds for character sums and extended the range of the subject by studying irregularities of distribution with respect to more general sequences.

Let N be a positive integer and let $\varepsilon_1, \ldots, \varepsilon_N$ be elements of $\{1, -1\}$. Put $\varepsilon_i = 0$ for i less than 1 or greater than N. In 1964 Roth [55] proved that there exist positive numbers c_0 and c_1 such that if N exceeds c_0 then

$$\max_{a,q,t \in \mathbb{Z}^+} \left| \sum_{j=1}^{t} \varepsilon_{a+jq} \right| > c_1 N^{1/4}.$$

Ten years later Sárközy [58] proved that there exist sequences for which the above maximum is appreciably smaller than $N^{1/2}$, an estimate which one can obtain by examining random sequences. He proved that there exists a positive number c_2 such that for each positive integer N (> 1) there exists a sequence $(\varepsilon_1, \ldots, \varepsilon_N)$ for which the maximum is at most $c_2(N \log N)^{1/3}$. In 1981 Beck [9] improved this to $c_3 N^{1/4} (\log N)^{5/2}$ and in 1996 Matoušek and Spencer [45] showed that Roth's result was essentially best possible by showing that there exist sequences for which the maximum is at most $c_4 N^{1/4}$.

As for irregularities of distribution with respect to other sequences, we note that an arithmetical progression is a shift and a dilation of the sequence of positive integers. Let r be a positive integer. In 1999 Beck, Sárközy and Stewart [10] proved that if N exceeds 5^{r+1} then

$$\max_{a \in \mathbb{Z},\ q,t \in \mathbb{Z}^+} \left| \sum_{j=1}^{t} \varepsilon_{a+j^r q} \right| \geq \frac{1}{4} N^{1/2(r+1)}.$$

Thus, no matter how we partition the first N (≥ 125) integers into two sets, there will be a shift and dilation of the initial terms of the sequence of squares which contains at least $(N^{1/6})/4$ more terms from one set than the other.

Let N be a positive integer and let $A = (a_n)_{n=1}^{\infty}$ be an increasing sequence of positive integers. Let $A(N)$ denote the number of terms of A which are at most N. Lovász conjectured that if

$$\overline{\lim} \frac{A(N)}{N} > 0,$$

then there exist i, j with $i \neq j$ such that $a_i - a_j$ is the square of an integer. András proved [60] Lovász' conjecture by means of the Hardy–Littlewood method as elaborated by Roth in his work on sets of integers which do not contain three term arithmetical progressions. Furstenberg [36] gave another proof of Lovász' conjecture using ergodic theory. In fact, András proved a quantitative refinement of Lovász' conjecture. He showed that if there is no difference which is a square then

$$\frac{A(N)}{N} = O\left(\frac{(\log\log N)^{2/3}}{(\log N)^{1/3}}\right).$$

This has since been sharpened by Pintz, Steiger and Szemerédi [53] who proved that

$$\frac{A(N)}{N} = O\left((\log N)^{-(\log\log\log\log N)/12}\right).$$

András also addressed the more difficult problem of showing that if A has the property that $a_i - a_j$ is never of the form $p-1$ with p a prime then A has upper density 0. Note that if instead of the primes shifted by 1 we took the primes we would be in a different situation since the positive integers divisible by 4 have the property that all their differences are divisible by 4. András proved in 1978 [61] that

$$\frac{A(N)}{N} = O\left(\frac{(\log_3 N)^3 \log_4 N}{(\log_2 N)^2}\right);$$

where $\log_1 N = \log N$ and $\log_i N = \log(\log_{i-1} N)$ for $i = 2, 3, \ldots$. We remark that Kamae and Mendès-France [40] in 1978 gave a general criterion for determining when a set of positive integers has the property that all sets of positive upper density possess a difference from that set.

For any integer n larger than one let $P(n)$ denote the greatest prime factor of n. Fujii [35], Erdős and Balog and Sárközy [6] all conjectured that every sufficiently large integer can be written as the sum of two smooth numbers.

CONJECTURE. *For each $\varepsilon > 0$ there exists a positive number N_0, which depends on ε, such that if N is a positive integer which exceeds N_0 then there exist positive integers n_1 and n_2 with*

$$n_1 + n_2 = N \qquad \textit{and} \qquad P(n_1 n_2) < N^{\varepsilon}.$$

As a step in the direction of the above conjecture Balog and Sárközy [6] proved that there is a positive number N_1 such that if N is a positive integer which exceeds N_1 then there exist positive integers n_1 and n_2 with

$$n_1 + n_2 = N \qquad \text{and} \qquad P(n_1 n_2) \le 2N^{2/5},$$

and later Balog improved this result.

If more summands are allowed then a much stronger conclusion applies. In 1981 Fujii [35] proved that the analogue of the above conjecture holds when 3 summands are used. Three years later Balog and Sárközy [5] proved that there is a positive number N_2 such that if N is a positive integer which exceeds N_2 then there exist positive integers n_1, n_2 and n_3 with

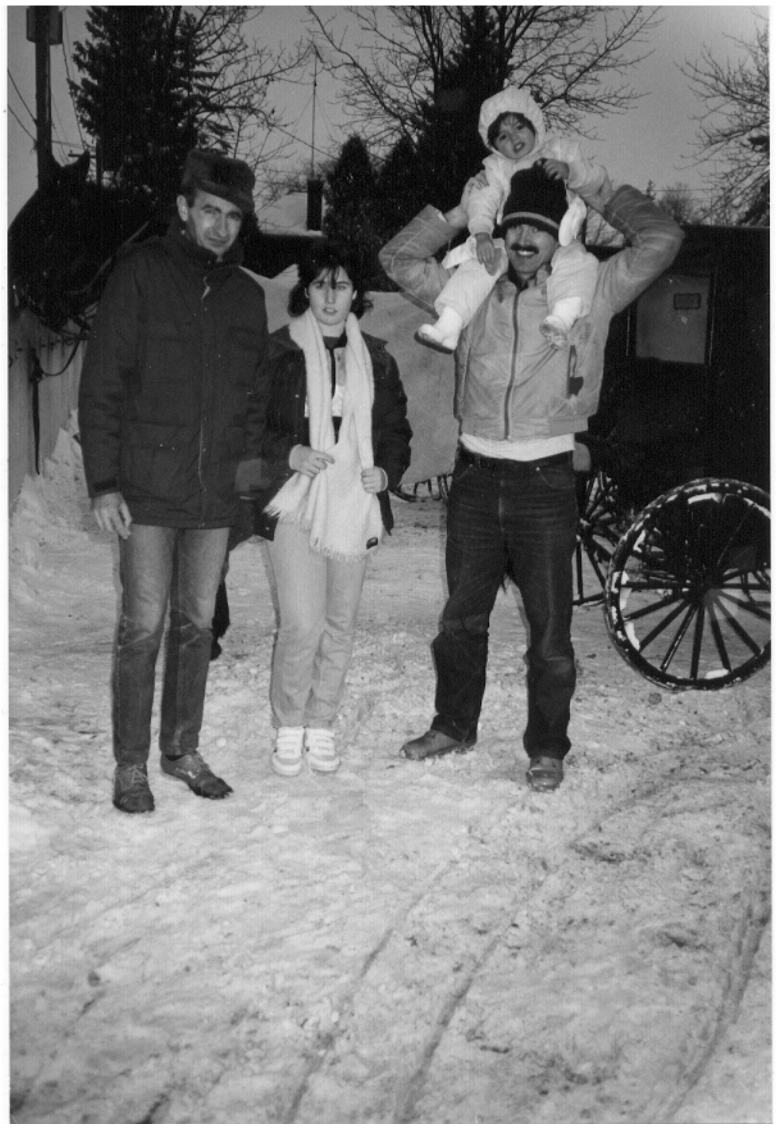
$$n_1 + n_2 + n_3 = N$$

and

$$P(n_1 n_2 n_3) \leq \exp(3(\log N \log_2 N)^{1/2}).$$

The proof of this result makes use of the circle method.

This brings me to my first picture which was taken in December of 1985 during a visit of András and his daughter Andrea to Waterloo. My daughter Elisa is on my shoulders.



A few years later Iwaniec and Sárközy [39] addressed the following problem. Let N be a positive integer and let A and B be subsets of $\{N, N+1, \ldots, 2N\}$. For any set X let $|X|$ denote its cardinality. Suppose that $|A| >> N$ and $|B| >> N$. How close is a product ab to a square? They proved, by means of an estimate for weighted exponential sums at well spaced points due to Bombieri and Iwaniec, that there exists an integer a from A, an integer b from B and a positive integer c such that

$$|ab - c^2| << (c \log c)^{1/2}.$$

An arithmetical function f is said to be multiplicative if, whenever m and n are coprime positive integers,

$$f(mn) = f(m)f(n). \tag{1}$$

It is strictly multiplicative if (1) holds for all positive integers m and n. We say that f satisfies a linear recurrence of finite order if there is a positive integer k and complex numbers $a_0, \ldots, a_k$ with $a_0 \neq 0$ and $a_k \neq 0$ such that

$$a_0 f(n) + a_1 f(n+1) + \cdots + a_k f(n+k) = 0,$$

for $n = 1, 2, \ldots$.

Sárközy [62] determined all multiplicative functions f which satisfy a linear recurrence of finite order. They are, in general, of the form $n^h \chi(n)$ where h is a non-negative integer and χ is a character. Let A^* denote the set of strictly multiplicative arithmetical functions f which satisfy a linear recurrence of finite order, are not identically zero, and satisfy $f(n) = o(n)$. He proved that f is in A^* if and only if there is a positive integer m such that f is a character modulo m. (Earlier Lovász, Sárközy and Simonovits [42] characterized additive arithmetical functions satisfying a linear recurrence of finite order.)

In a series of papers András investigated the arithmetical character of sumsets. Let N be a positive integer and let A and B be subsets of $\{1, \ldots, N\}$. In 1984 Balog and Sárközy [7] proved, by means of the large sieve inequality, that if

$$|A| >> N \quad \text{and} \quad |B| >> N \tag{2}$$

then there exist a in A and b in B with

$$P(a+b) >> N/\log N.$$

They also proved [8], under the same assumption, that there exist a_1 in A and b_1 in B and a prime p such that $p^2 \mid a_1 + b_1$ and

$$p^2 >> N/(\log N)^7.$$

For the proof they employed the circle method. In 1986 and 1988 Sárközy and Stewart [65, 67] used the circle method to sharpen these results. They proved that if k is a positive integer and (2) holds, then there exist a in A and b in B and a prime p such that p^k divides $a+b$ and

$$p^k >>_k N. \tag{3}$$

In 1992 Ruzsa [56] gave a new proof of (3) for the case when $k = 1$.

The general philosophy behind results of this sort is that if A and B are sufficiently dense subsets of $\{1, \dots, N\}$ then arithmetical properties of the sums $a+b$ should mirror those of the first $2N$ integers. With this in mind, it is reasonable to ask if an Erdős–Kac theorem holds for the sums $a+b$. In 1987 Erdős, Maier and Sárközy [16] established such a result. For any positive integer n let $\omega(n)$ denote the number of prime factors of n. They proved that

$$\frac{1}{|A||B|}\left|\left\{(a,b) : \frac{\omega(a+b) - \log\log N}{(\log\log N)^{1/2}} < x,\ a \in A,\ b \in B\right\}\right|$$

is asymptotic to

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-u^2/2}du,$$

provided that

$$\frac{|A||B|}{N^2/(\log\log N)^{1/2}} \to \infty \quad \text{as } N \to \infty.$$

Both Elliott and Sárközy [15] and Tenenbaum [69] have extended this result.

While the above results treat the average behaviour of $\omega(a+b)$ one may also investigate the extreme values assumed by $\omega(a+b)$. If (2) holds then it follows from (3) that

$$\min_{a\in A,\ b\in B} \omega(a+b) = O(1).$$

What can be said about large values of $\omega(a+b)$?

Let $m(N)$ denote the largest integer m for which the product of the first m primes is at most N. Thus

$$m(N) = \max\{\omega(k) : k \le N\}.$$

In 1993 Erdős, Pomerance, Sárközy and Stewart [17] proved the following. For any real number x let $[x]$ denote the greatest integer less than or equal to x. Let ε be a positive real number and suppose that A and B are subsets of $\{1, \dots, [N/2]\}$ with

$$|A||B| > \varepsilon N^2.$$

There exist positive numbers $c(\varepsilon)$ and $N_0(\varepsilon)$, which depend on ε such that if N exceeds $N_0(\varepsilon)$ then there exist a in A and b in B with

$$\omega(a+b) > m(N) - c(\varepsilon)\sqrt{m(N)}. \tag{4}$$

Further (4) is best possible up to replacing $\sqrt{m(N)}$ by $\sqrt{m(N)}/\log m(N)$. One of the ingredients in the proof is a result of Katona on the intersection of subsets of a set.

This brings me to the next two pictures. We were working on the above paper in the winter of 1991. András and Carl Pomerance were both visiting Waterloo and we decided to do some ice fishing. Picture 2 shows András on the ice of Lake Simcoe surrounded by ice fishing huts. Picture 3 shows Carl and András fishing in one of the huts. Alas, our catch that day was the empty set.

Let $A = \{a_1, a_2, \dots\}$ be a set of positive integers and put $\overline{A} = \mathbb{Z}^+ \backslash A$. As before, for N in $\mathbb{Z}^+$ put

$$A(N) = |A \cap \{1, \dots, N\}|$$

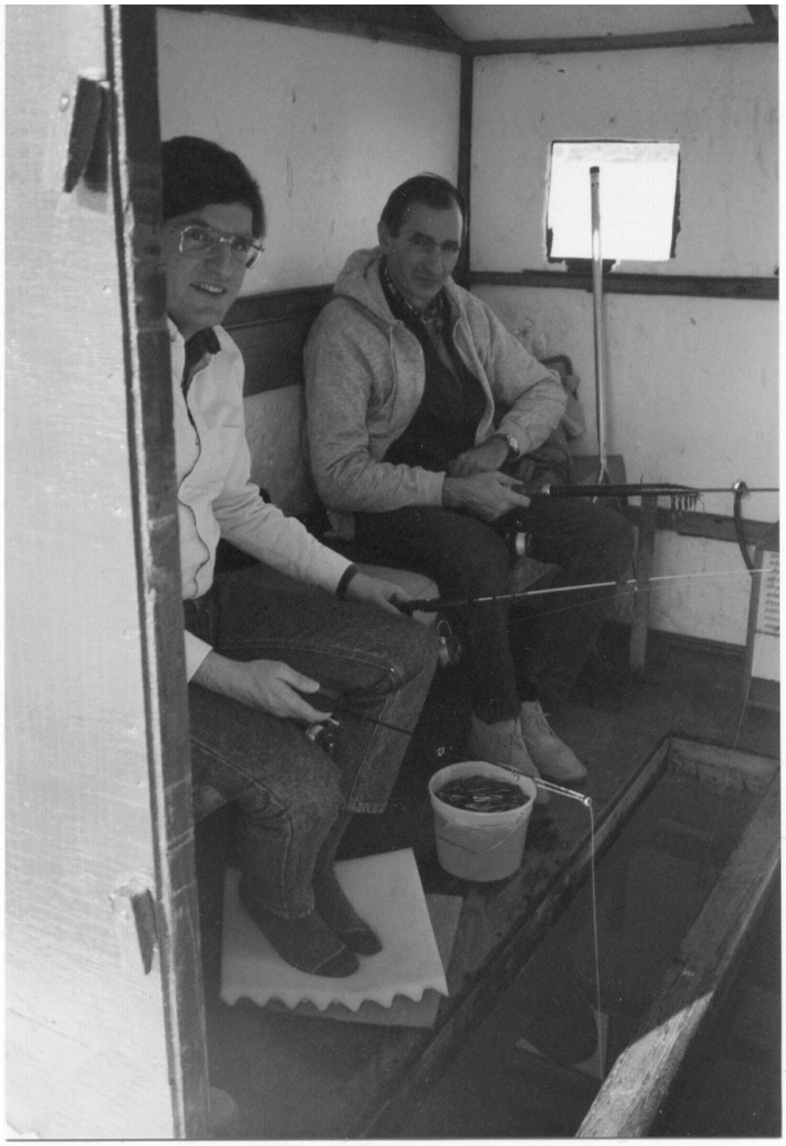
and

$$\overline{A}(N) = |\overline{A} \cap \{1, \dots, N\}|.$$

For each positive integer n let $R(n)$ $(= R_A(n))$ denote the number of solutions of

$$a_i + a_j = n \quad \text{with } i \le j.$$

Suppose $a_1 < a_2 < \dots$. In a series of papers [19, 20, 21, 22, 23] Erdős, Sárközy and Sós investigated the behaviour of the function $R(n)$ and two related counting functions. They proved that any smooth function in the appropriate range can be



realized as a counting function R_A for some set A up to a small error and they gave limitations on the size of the error. They also studied when $R(n)$ is monotone and when $R(n) - R(n-1)$ is bounded. For instance they proved that if

$$\lim_{N\to\infty} \frac{\overline{A}(N)}{\log N} = \infty$$

then there is no positive number C such that

$$R(n+1) \ge R(n) \quad \text{for } n > C.$$

Balasubramanian [4] gave an independent proof of this result. Further, A is said to be a Sidon sequence if $R(n)$ is at most 1 for all positive integers n. They proved that if A is an infinite Sidon sequence then there exist infinitely many integers k for which $R(2k) = 1$ and $R(2k+1) = 0$.

In 1992 [64] András proposed the study of the arithmetical character of integers of the form $ab+1$ where a is from A and b is from B where A and B are dense subsets of the first N integers. In this case the expectation is that the properties of the terms $ab+1$ should be similar to those of the first N^2+1 positive integers and results of this sort have been established. Furthermore, just as in the additive case, it is possible to prove some results when no restriction is placed on the density of the sets A and B. For instance, in 1996, Győry, Sárközy and Stewart [38] proved that if A and B are finite sets of positive integers with $|A| \ge |B| \ge 2$ then there exists an effectively computable positive number c such that

$$\omega\left(\prod_{a\in A,\ b\in B} (ab+1)\right) > c\log|A|.$$

The proof relies on estimates for the number of solutions of S-unit equations due to Evertse. In addition, they proved that there are large sets A and B for which all expressions of the form $ab+1$ have small prime factors. Let ε be a positive real number and suppose that k and ℓ are positive integers with

$$k \ge 16 \quad \text{and} \quad 2 \le \ell \le \left(\frac{\log_2 k}{\log_3 k}\right)^{1/2}.$$

There exists a positive number $C(\varepsilon)$, which depends on ε, such that if k exceeds $C(\varepsilon)$ then there are sets of positive integers A and B with $|A| = k$ and $|B| = \ell$ for which

$$P\left(\prod_{a\in A}\prod_{b\in B}(ab+1)\right) < (\log k)^{\ell+1+\varepsilon}.$$

The additive analogue of this result is due to Erdős, Stewart and Tijdeman [34].

In this context Győry, Sárközy and Stewart [38] made the following conjecture.

Conjecture. *Let a, b and c denote distinct positive integers. If* $\max(a,b,c) \to \infty$ *then*

$$P((ab+1)(bc+1)(ca+1)) \to \infty.$$

While the conjecture remains open, Győry and Sárközy [37] proved in 1997 that it holds if at least one of $a, b, c, \frac{a}{b}, \frac{b}{c}$ and $\frac{c}{a}$ has bounded prime factors. Bugeaud [12] later gave a quantitative form of this result. Also in 1997 Stewart and Tijdeman [68] showed that the conjecture holds if $(\log a)/\log(c+1)$ tends to infinity.

For any positive integer n let $p(n)$ denote the number of partitions of n. In papers from 1921 and 1926 MacMahon [43, 44] developed efficient methods for calculating the parity of $p(n)$. During this period Ramanujan determined congruences that hold for $p(n)$ with respect to other moduli such as 5, 7 and 11. However it was not known that $p(n)$ is even for infinitely many integers n or that $p(n)$ is odd for infinitely many integers n until the work of Kolberg [41] in 1959. Mirsky [48], in 1983, was the first to give a quantitative form of Kolberg's result. His result was sharpened by Nicolas and Sárközy [50] in 1995. They proved that there exist positive numbers C and c such that if N exceeds C then

$$|\{n : p(n) \equiv 0 (\bmod\ 2),\ n \le N\}| > (\log N)^c$$

and

$$|\{n : p(n) \equiv 1 (\bmod\ 2),\ n \le N\}| > (\log N)^c.$$

A significant improvement of these estimates was obtained by Nicolas, Ruzsa and Sárközy [49] three years later. They proved that there exist positive numbers C_1 and C_2 such that if N exceeds C_1 then there are at least $C_2 N^{1/2}$ positive integers n up to N for which $p(n)$ is even. Further, for each positive real number ε there is a number $C_3(\varepsilon)$ such that if N exceeds $C_3(\varepsilon)$ then there are at least

$$N^{1/2} \exp(-(\log 2 + \varepsilon) \log N / \log\log N) \tag{5}$$

positive integers n up to N for which $p(n)$ is odd. The proofs of these results are elementary in character. Serre [49] gave a different proof of the first assertion by making quantitative a theorem of Ono [51]. The proof relies on the theory of modular forms. Ahlgren [1] gave a quantitative version of Ono's theorem for odd values of the partition function and was able to sharpen (5) to $cN^{1/2}/\log N$ for some positive number c, see also Ono [52].

The final aspect of András' work that I wish to discuss concerns finite pseudorandom binary sequences. Let N be a positive integer and let $E_N = (\varepsilon_1, \dots, \varepsilon_N)$ be a binary sequence with ε_i in $\{-1, 1\}$ for $i = 1, \dots, N$. Put

$$U(E_N, t, a, b) = \sum_{j=1}^{t} \varepsilon_{a+jb}.$$

The well distribution measure $W(E_N)$ of E_N is given by

$$W(E_N) = \max_{a,b,t} |U(E_N, t, a, b)|$$

where the maximum is taken over all triples (a, b, t) with a an integer, b and t

positive integers and $1 \le a+b \le a+tb \le N$. Next let k and M be positive integers and let $D = (d_1, \dots, d_k)$ with $d_1, \dots, d_k$ integers satisfying $0 \le d_1 < \cdots < d_k$. Put

$$V(E_N, M, D) = \sum_{n=1}^{M} \varepsilon_{n+d_1} \cdots \varepsilon_{n+d_k}.$$

The correlation measure $C_k(E_N)$ of order k of E_N is defined as

$$C_k(E_N) = \max_{M,D} |V(E_N, M, D)|,$$

where the maximum is taken over all D and M such that $M + d_k$ is at most N. Mauduit and Sárközy [46] introduced the well distribution measure and the correlation measure as tests for randomness. The objective is to find sequences for which $W(E_N)$ and $C_k(E_N)$ are small for small positive integers k. Such sequences are called pseudorandom. Niederreiter had earlier introduced a different measure of randomness based on ideas from the theory of uniform distribution.

Mauduit and Sárközy [46] proved that if p is a prime number and $N = p - 1$ then the sequence

$$E_N = \left\{ \left(\frac{1}{p}\right), \left(\frac{2}{p}\right), \dots, \left(\frac{N}{p}\right) \right\},$$

where $\left(\frac{i}{p}\right)$ is the Legendre symbol for i modulo p, satisfies

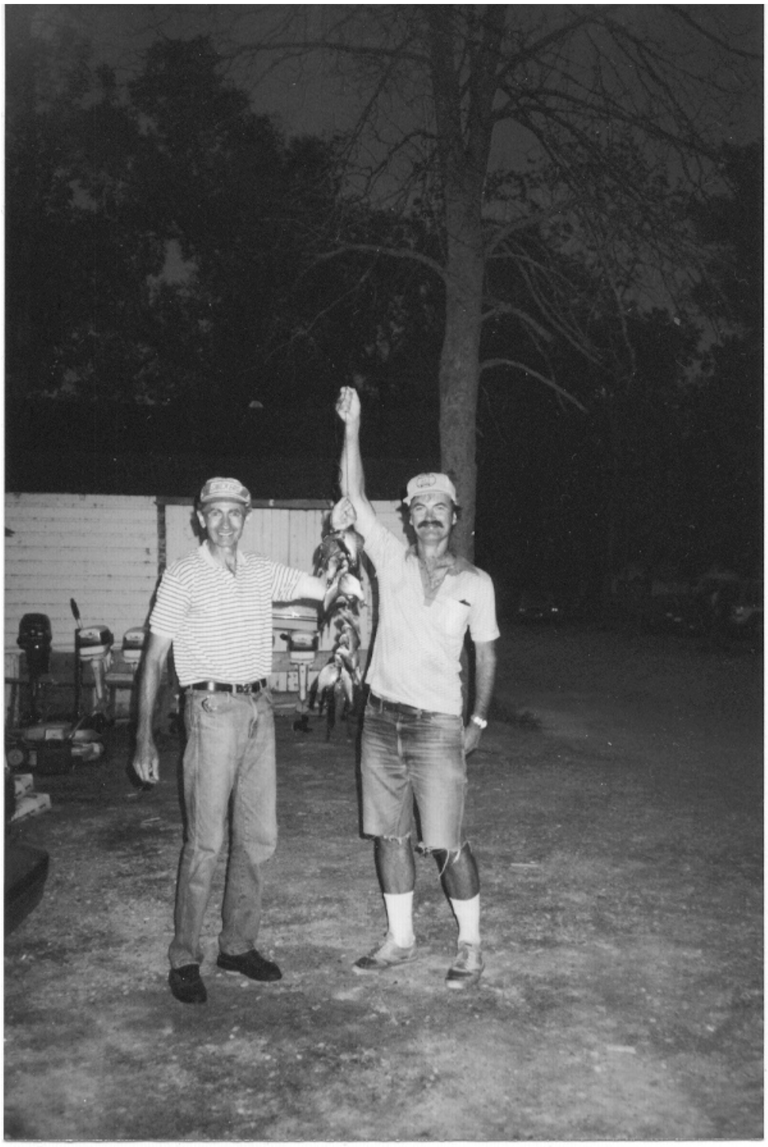
$$W(E_N) << N^{1/2} \log N \quad \text{and} \quad C_k(E_N) << kN^{1/2} \log N,$$

and so is a “good” pseudorandom sequence. The proof of these results depends on Weil’s theorem. Mauduit and Sárközy [47] also gave estimates for $W(E_N)$ and $C_k(E_N)$ when E_N is an initial segment of such well known sequences as the Champernowne sequence, the Rudin–Shapiro sequence and the Thue–Morse sequence. In addition Cassaigne, Ferenczi, Mauduit, Rivat and Sárközy [13] studied a sequence generated by the Liouville function λ. For each positive integer n let $\Omega(n)$ denote the number of prime factors of n counted with multiplicity. The Liouville function is $\lambda(n) = (-1)^{\Omega(n)}$. Put

$$L_N = (\lambda(1), \dots, \lambda(N)).$$

While the expectation is that the sequence behaves like a random sequence, this is very difficult to show. They were able to show, subject to the generalized Riemann hypothesis, that for each positive number ε there is a positive number $N_1(\varepsilon)$ such that if N exceeds $N_1(\varepsilon)$ then

$$W(L_N) < N^{5/6+\varepsilon}.$$



Let me conclude with a picture showing that some of our fishing expeditions have been successful and with a wish for András on his sixtieth birthday that he catch many more tasty fish and that he prove many more great theorems.

REFERENCES

[1] S. Ahlgren, Distribution of parity in the partition function on arithmetic progressions, *Indag. Math.* **10** (1999), 173–181.

[2] R. Ahlswede, L. H. Khachatrian and A. Sárközy, On the counting function of primitive sets of integers, *J. Number Theory* **79** (1999), 330–344.

[3] R. Ahlswede, L. H. Khachatrian and A. Sárközy, On the quotient sequence of sequences of integers, *Acta Arith.* **91** (1999), 117–132.

[4] R. Balasubramanian, A note on a result of Erdős, Sárközy and Sós, *Acta Arith.* **49** (1987), 45–53.

[5] A. Balog and A. Sárközy, On sums of integers having small prime factors, I., *Studia Sci. Math. Hungar.* **19** (1984), 35–47.

[6] A. Balog and A. Sárközy, On sums of integers having small prime factors, II., *Studia Sci. Math. Hungar.* **19** (1984), 81–89.

[7] A. Balog and A. Sárközy, On sums of sequences of integers, II., *Acta Math. Acad. Sci. Hungar.* **44** (1984), 169–179.

[8] A. Balog and A. Sárközy, On sums of sequences of integers, III., *Acta Math. Acad. Sci. Hungar.* **44** (1984), 339–349.

[9] J. Beck, Roth's estimate on the discrepancy of integer sequences is nearly sharp, *Combinatorica* **1** (1981), 319–325.

[10] J. Beck, A. Sárközy and C. L. Stewart, On irregularities of distribution in shifts and dilations of integer sequences, II., in: *Number Theory in Progress*, Proceedings of the International Conference on Number Theory organized by the Stefan Banach International Mathematical Center in honor of the 60$^{\text{th}}$ birthday of Andrzej Schinzel, Zakopane, Poland, June 30–July 9, 1997, eds. K. Győry et al., Walter de Gruyter, Berlin-New York, 1999; 633–638.

[11] F. Behrend, On sequences of integers not divisible one by another, *J. London Math. Soc.* **10** (1935), 42–44.

[12] Y. Bugeaud, On the greatest prime factor of $(ab+1)(bc+1)(ca+1)$, *Acta Arith.* **86** (1998), 45–49.

[13] J. Cassaigne, S. Ferenczi, C. Mauduit, J. Rivat and A. Sárközy, On finite pseudorandom binary sequences, III. (The Liouville function, I.), *Acta Arith.* **87** (1999), 367–390.

[14] Á. Elbert and A. Sárközy, Über rationale Polynome, *Annales Univ. Sci. Budapest Eötvös* **5** (1962), 155–171.

[15] P. D. T. A. Elliott and A. Sárközy, The distribution of the number of prime divisors of sums $a+b$, *J. Number Theory* **29** (1988), 94–99.

[16] P. Erdős, H. Maier and A. Sárközy, On the distribution of the number of prime factors of sums $a+b$, *Trans. Amer. Math. Soc.* **302** (1987), 269–280.

[17] P. Erdős, C. Pomerance, A. Sárközy and C. L. Stewart, On elements of sumsets with many prime factors, *J. Number Theory* **44** (1993), 93–104.

[18] P. Erdős and A. Sárközy, On the divisibility properties of sequences of integers, *Proc. London Math. Soc.* **21** (1970), 97–101.

[19] P. Erdős and A. Sárközy, Problems and results on additive properties of general sequences, I., *Pacific J. Math.* **118** (1985), 347–357.

[20] P. Erdős and A. Sárközy, Problems and results on additive properties of general sequences, II., *Acta Math. Acad. Sci. Hungar.* **48** (1986), 201–211.

[21] P. Erdős, A. Sárközy and V. T. Sós, Problems and results on additive properties of general sequences, IV., in: *Number Theory*, Proceedings, Ootacamund, India, 1984, ed. K. Alladi, *Lecture Notes in Mathematics* **1122**, Springer-Verlag, Berlin–Heidelberg–New York, 1985; 85–104.

[22] P. Erdős, A. Sárközy and V. T. Sós, Problems and results on additive properties of general sequences, V., *Monatshefte Math.* **102** (1986), 183–197.

[23] P. Erdős, A. Sárközy and V. T. Sós, Problems and results on additive properties of general sequences, III., *Studia Sci. Math. Hungar.* **22** (1987), 53–63.

[24] P. Erdős, A. Sárközy and E. Szemerédi, On divisibility properties of sequences of integers, *Studia Sci. Math. Hungar.* **1** (1966), 431–435.

[25] P. Erdős, A. Sárközy and E. Szemerédi, On the divisibility properties of sequences of integers, I., *Acta Arith.* **11** (1966), 411–418.

[26] P. Erdős, A. Sárközy and E. Szemerédi, On the solvability of the equations $[a_i, a_j] = a_r$ and $(a'_i, a'_j) = a'_r$ in sequences of positive density, *J. Math. Anal. Appl.* **15** (1966), 60–64.

[27] P. Erdős, A. Sárközy and E. Szemerédi, On a theorem of Behrend, *J. Australian Math. Soc.* **7** (1967), 9–16.

[28] P. Erdős, A. Sárközy and E. Szemerédi, On an extremal problem concerning primitive sequences, *J. London Math. Soc.* **42** (1967), 484–488.

[29] P. Erdős, A. Sárközy and E. Szemerédi, The solvability of certain equations in dense sequences of integers (in Russian), *Dokl. Akad. Nauk SSSR* **176** (1967), 541–544.

[30] P. Erdős, A. Sárközy and E. Szemerédi, On the divisibility properties of sequences of integers, II., *Acta Arith.* **14** (1968), 1–12.

[31] P. Erdős, A. Sárközy and E. Szemerédi, On the solvability of certain equations in sequences of positive upper logarithmic density, *J. London Math. Soc.* **43** (1968), 71–78.

[32] P. Erdős, A. Sárközy and E. Szemerédi, Über Folgen ganzer Zahlen, in: *Abhandlungen aus Zahlentheorie und Analysis* (zur Errinnerung an E. Landau), Plenum, New York, 1969; 77–86.

[33] P. Erdős, A. Sárközy and E. Szemerédi, On divisibility properties of sequences of integers, *Coll. Math. Soc. J. Bolyai* **2** (Number Theory, Debrecen, 1968) (1970), 35–49.

[34] P. Erdős, C. L. Stewart and R. Tijdeman, Some Diophantine equations with many solutions, *Compositio Math.* **66** (1988), 37–56.

[35] A. Fujii, An additive problem in theory of numbers, *Acta Arith.* **40** (1981), 41–49.

[36] H. Furstenberg, Ergodic behaviour of diagonal measures and a theorem of Szemerédi on arithmetic progressions, *J. d'Analyse Math.* **31** (1977), 204–256.

[37] K. Győry and A. Sárközy, On prime factors of integers of the form $(ab+1)(bc+1)(ca+1)$, *Acta Arith.* **79** (1997), 163–171.

[38] K. Győry, A. Sárközy and C. L. Stewart, On the number of prime factors of integers of form $ab+1$, *Acta Arith.* **74** (1996), 365–385.

[39] H. Iwaniec and A. Sárközy, On a multiplicative hybrid problem, *J. Number Theory* **26** (1987), 89–95.

[40] T. Kamae and M. Mendès-France, Van der Corput's difference theorem, *Israel J. Math.* **31** (1978), 335–342.

[41] O. Kolberg, Note on the parity of the partition function, *Math. Scand.* **7** (1959), 377–378.

[42] L. Lovász, A. Sárközy and M. Simonovits, On additive arithmetic functions satisfying a linear recursion, *Annales Univ. Sci. Budapest Eötvös* **24** (1981), 205–215.

[43] P. A. MacMahon, Note on the parity of the number which enumerates the partitions of a number, *Proc. Cambridge Phil. Soc.* **20** (1921), 281–283.

[44] P. A. MacMahon, The parity of $p(n)$, the number of partitions of n, when $n \leq 1000$, *J. London Math. Soc.* **1** (1926), 225–226.

[45] J. Matoušek and J. Spencer, Discrepancy in arithmetic progressions, *J. Amer. Math. Soc.* **9** (1996), 195–204.

[46] C. Mauduit and A. Sárközy, On finite pseudorandom binary sequences, I. Measure of pseudorandomness, the Legendre symbol, *Acta Arith.* **82** (1997), 365–377.

[47] C. Mauduit and A. Sárközy, On finite pseudorandom binary sequences, II. (The Champernowne, Rudin–Shapiro and Thue–Morse sequences. A further construction.), *J. Number Theory* **73** (1998), 256–276.

[48] L. Mirsky, The distribution of the partition function in residue classes, *J. Math. Anal. Appl.* **93** (1983), 593–598.

[49] J.-L. Nicolas, I. Z. Ruzsa and A. Sárközy, On the parity of additive representation functions, *J. Number Theory* **73** (1998), 292–317.

[50] J.-L. Nicolas and A. Sárközy, On the parity of partition functions, *Illinois J. Math.* **39** (1995), 586–597.

[51] K. Ono, Parity of the partition function in arithmetic progressions, *J. Reine Angew. Math.* **472** (1996), 1–15.

[52] K. Ono, Distribution of the partition function modulo m, *Ann. Math.* **151** (2000), 293–307.

[53] J. Pintz, W. L. Steiger and E. Szemerédi, On sets of natural numbers whose difference set contains no squares, *J. London Math. Soc.* **37** (1988), 219–231.

[54] C. Pomerance and A. Sárközy, On homogeneous multiplicative hybrid problems in number theory, *Acta Arith.* **49** (1988), 291–302.

[55] K. F. Roth, Remark concerning integer sequences, *Acta Arith.* **9** (1964), 257–260.

[56] I. Ruzsa, Large prime factors of sums, *Studia Sci. Math. Hungar.* **27** (1992), 463–470.

[57] A. Sárközy, Some remarks concerning irregularities of distribution of sequences of integers in arithmetic progressions, I., *Coll. Math. Soc. J. Bolyai* **13** (Topics in Number Theory, Debrecen) (1974), 287–303.

[58] A. Sárközy, Some remarks concerning irregularities of distribution of sequences of integers in arithmetic progressions, II., *Studia Sci. Math. Hungar.* **11** (1976), 79–103.

[59] A. Sárközy, Some remarks concerning irregularities of distribution of sequences of integers in arithmetic progressions, IV., *Acta Math. Acad. Sci. Hungar.* **30** (1977), 155–162.

[60] A. Sárközy, On difference sets of sequences of integers, I., *Acta Math. Acad. Sci. Hungar.* **31** (1978), 125–149.

[61] A. Sárközy, On difference sets of sequences of integers, III., *Acta Math. Acad. Sci. Hungar.* **31** (1978), 355–386.

[62] A. Sárközy, On multiplicative arithmetic functions satisfying a linear recursion, *Studia Sci. Math. Hungar.* **13** (1978), 79–104.

[63] A. Sárközy, Some remarks concerning irregularities of distribution of sequences of integers in arithmetic progressions, III., *Periodica Math. Hungar.* **9** (1978), 127–144.

[64] A. Sárközy, On sums $a+b$ and numbers of the form $ab+1$ with many prime factors, *Grazer Math. Ber.* **318** (1992), 141–154.

[65] A. Sárközy and C. L. Stewart, On divisors of sums of integers, II., *J. Reine Angew. Math.* **365** (1986), 171–191.

[66] A. Sárközy and C. L. Stewart, On irregularities of distribution in shifts and dilations of integer sequences, I., *Math. Annalen* **276** (1987), 353–364.

[67] A. Sárközy and C. L. Stewart, On divisors of sums of integers, IV., *Canadian J. Math.* **40** (1988), 788–816.
[68] C. L. Stewart and R. Tijdeman, On the greatest prime factor of $(ab+1)(ac+1)(bc+1)$, *Acta Arith.* **79** (1997), 93–101.
[69] G. Tenenbaum, Facteurs premiers de sommes d'entiers, *Proc. Amer. Math. Soc.* **106** (1989), 287–296.

C. L. Stewart
Department of Pure Mathematics
University of Waterloo
Waterloo, Ontario
Canada N2L 3G1
E-mail: cstewart@watserv1.uwaterloo.ca