

## Algebra Exam Syllabus

### Linear Algebra

Jordan and rational canonical form for matrices; Inner product spaces; Spectral theorem; Hermitian, unitary, and normal operators; Properties of positive definite matrices and operators; Dual space and the transpose of an operator; Tensor products of vector spaces.

### Groups and Rings

Isomorphism Theorems; Jordan-Hölder series; Krull-Schmidt Theorem; Sylow Theorems; Free abelian groups; Structure of finitely generated abelian groups; Divisible abelian groups; Symmetric and Alternating groups; Central series and nilpotent groups; Solvable groups; Free groups; Presentation of groups; Linear groups. Principal ideal domains, Euclidean domains; unique factorization domains; Chinese remainder theorem; Finitely generated modules over PID's; Noetherian and Artinian modules; Polynomial rings and Hilbert's Basis Theorem.

### Fields and Galois

Algebraic extensions; Algebraic closure; Splitting fields of polynomials; Separable and inseparable extensions; Simple extensions; Galois correspondence; Norms and traces; Finite fields; Equations solvable by radicals; Transcendental extensions and transcendence degree; Algebraic number fields.

### References

1. D. Dummit and R. Foote, Abstract Algebra, 1991.
2. J. Rotman, Advanced Modern Algebra, 2002.
3. M. Artin, Algebra, 1991. 4. T. Hungerford, Algebra, 2003