Analysis and Topology Exam Syllabus

Topology and Real Analysis

Continuity, properties of continuous functions [HM 4.1-4.7, BBT ch 5, Rud ch 4]
Pointwise and uniform convergence [HM 5.1-5.3, BBT ch 9, 10, 13.5, Rud 7.2-7.5]
Metric spaces – topology, compactness, completeness, connectedness, [HM ch 2-3, BBT 13.1-13.8, 13.12, Rud ch 2, Roy 7.7]
C(X) for X compact, Hausdorff space or complete metric space, Arzela- Ascoli theorem, Stone Weierstrass theorem [HM 5.5-5.6, 5.8, Rud 7.6-7.7, Roy 7.10, 9.9]
Banach contraction mapping principle [HM 5.7, BBT 13.9-13.11]
Baire category theorem [HM p.175, BBT 13.13, Roy 7.8]
Axiom of Choice and its equivalents, transfinite induction [H 14-18]
Cardinal and ordinal numbers, Schroeder-Bernstein theorem [H 19-25]

Complex Analysis

Analytic functions, Cauchy-Riemann equations [A 2.1, 3.2, C III.2-3]
Cauchy's theorems and the Cauchy integral formula, open mapping theorem [A 4.1-2, 4.4, C IV.4-7] Liouville's theorem [A 4.2, C IV.3]
Maximum modulus principle, Schwarz Lemma [A 4.3, C IV.3, VI.1-2]
Laurent series, Analytic continuation [A 4.3, 5.1, 8.1, C III.1, IV.2, V.1, IX.1-3]
Meromorphic functions, Rouche's theorem [A 4.3, C 5.3]
Residue theorem and its applications, Contour integrals [A 4.5, C V.2]
Harmonic functions, Normal families [A. 4.6, 5.5, 6.3, C VII.1-3, X.1-2]
Riemann mapping theorem, Conformal mappings [A 6.1-2, C VII.4]
Picard's theorem [A 8.3, C XII.1-4]

Measure Theory and Fourier Analysis

Lebesgue measure and the Lebesgue integral, the Lebesgue convergence theorems [HM 8.1-8.4, 8.6, Rud ch 11, Roy ch 3,4]
L^p and l^p spaces, Holder's inequality [Roy ch 6, BBT2 13.1-13.3]
Fubini's theorem [HM 9.2, Roy 12.4, BBT2 6.1-6.3]
Fourier series, Parseval's theorem, Dirichlet and Fejer kernels, convergence theorems. [HM 10.2-10.6, BBT2 15.1-15.9]
Hilbert spaces, orthogonality, basis, separability, duality [HM 10.1-2, BBT2 14.1-4, Roy 10.8, Rud 11.9]

References

[A] L. Alhlfors, Complex Analysis, 3rd Edition, McGraw-Hill, 1979.
[BBT] A. Bruckner, J. Bruckner and B. Thomson, Elementary real analysis, 2008 (Most of the book is at a lower level; it doesn't have the more advanced topics.)
[BBT2] A. Bruckner, J. Bruckner and B. Thomson, Real analysis, 1997 (Goes well beyond the exam material, but a good reference.)
[C] J.B. Conway, Functions of One Complex Variable I, 2nd Edition, Springer-Verlag, 1978.
[H] P.R. Halmos, Naive Set Theory, Springer-Verlag, 1974.
[L] S. Lang, Complex Analysis, 4th Edition, Springer-Verlag, 1999.
[HM] J. Marsden and M. Hoffman, Elementary classical analysis, 1993. (Almost everything is here.)
[MH] J. Marsden and M. Hoffman, Basic Complex Analysis, 3rd edition, W.H. Freeman, 1998.
[M] J.R. Munkres, Topology, 2nd ed., Prentrice Hall, 2000.
[Roy] H. Royden, Real analysis, 1988 (Covers Lebesgue integration very well.)
[Rud] W. Rudin, Principles of mathematical analysis, 1976. (It's a classic, but doesn't cover all of the more advanced topics.)
[W] S. Willard, General Topology, Dover, 2004.