PMATH 950 – Quantum Representation Theory, Winter 2023

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Lecture Time: Tuesdays and Thursdays 13:00-14:20
Location: MC 5479

Course Outline:

This course will serve as an introduction to the theory of compact quantum groups with a heavy emphasis on their representation theory. The general theory of quantum groups is a nowadays a vast branch of mathematics with applications in analysis, geometry, algebra, and physics. What makes quantum groups so fascinating is that one can approach the theory from various perspectives (e.g., analytic, algebraic, or categorical). In this course, we will take a mixed appproach to the subject, starting off by defining compact quantum groups using the functional analytic language of operator algebras, making connections later on to the algebraic approaches via Hopf algebras and deformed universal enveloping algebras of Lie algebras. Finally, towards the end of the course, we will see how compact quantum groups can, much like ordinary compact groups, be described by categorical data via their unitary representations. At the end of the course we will explore how all of the algebra, analysis, and category theory come together with some nice applications of quantum groups in quantum information theory.

Plan of Topics Covered:

(1) Introduction and Motivation: What is a (quantum) group? Why do we want to quantize groups? What's so quantum about quantum groups anyway?
(2) A brief overview of the main results on classical compact groups and their representation theory.
(3) A brief overview of some basic C*-algebra theory: Gelfand-Naimark theorem, universal C*-algebras, states, GNS construction.
(4) Operator algebraic compact quantum groups: Basic definitions, examples, properties, existence and uniqueness of Haar integrals, Hopf algebras and algebraic approaches to quantum groups.
(5) Representations of compact quantum groups, unitary representations, unitarizability, direct sums, tensor products, Schur's Lemma, complete reducibilty.
(6) Schur-Weyl orthogonality relations, quantum Peter-Weyl theory
(7) Recovering a quantum group from its representation theory: Representation categories, abstract tensor categories, the Tannaka-Krein reconstruction theorem.
(8) The Drinfeld-Jimbo q-deformed quantum groups.
(9) A selection of applications: Time permitting, we may discuss actions of quantum groups on classical/quantum spaces, quantum groups as symmetries in quantum physics, amenability, applications to subfactor theory.

Prerequisites: I will endeavour to make this course as accessible as possible to a broad range of students (e.g., those who may be interested in either algebra, analysis, or geometry). The language of operator algebras (e.g., material from PMATH 810) will be used throughout the course, however I will make an effort to quote the theorems and results we will use as the need arises. Students who have a basic familiarity with the following topics will get the most out of this course.

- **Groups and their representations**: The basic concepts from group theory and their representations on vector spaces.
- **Algebras**: Basic knowledge of what an (associative) algebra is, algebras defined by generators and relations, modules over algebras.

- **Functional Analysis**: Basic notions about measures and how they are viewed as linear functionals, basic C*-algebra theory (e.g., the material from PMATH 810).

Recommended Textbooks: There is no required textbook for this course. The lectures will be self-contained. Students may find the following optional textbooks useful. My lectures will sample material from all of these sources.

- S. Neshveyev and L. Tuset, *Compact Quantum Groups and Their Representation Categories*, Société Mathématique de France (2014). PDF available online via a Google search.
- T. Timmerman, *An Invitation to Quantum Groups and Duality: From Hopf Algebras to Multiplicative Unitaries and Beyond*, EMS Press (2008).
- A. Freslon, *Compact Quantum Groups and Their Combinatorics*, Cambridge Press (2022). This book is in press - a pdf copy will be made available to students in the course.

PMATH 950 Grading Scheme:	Class Attendance	50 %
	End of Term Presentation	50 %

The University Senate mandates that every course outline must contain the following text.

Academic Integrity: In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. Check

www.uwaterloo.ca/academicintegrity/

for more information.

Grievance: A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read *Policy 70, Student Petitions and Grievances, Section 4*,

http://www.adm.uwaterloo.ca/infosec/Policies/policy70.htm.

When in doubt please be certain to contact the department's administrative assistant who will provide further assistance.

Discipline: A student is expected to know what constitutes academic integrity to avoid committing academic offenses and to take responsibility for his/her actions. A student who is unsure whether an action constitutes an offense, or who needs help in learning how to avoid offenses (e.g., plagiarism, cheating) or about "rules" for group work/collaboration should seek guidance from the course professor, academic advisor, or the undergraduate associate dean. For information on categories of offenses and types of penalties, students should refer to *Policy 71, Student Discipline*,

http://www.adm.uwaterloo.ca/infosec/Policies/policy71.htm.

For typical penalties check *Guidelines for the Assessment of Penalties*,

http://www.adm.uwaterloo.ca/infosec/guidelines/penaltyguidelines.htm.

Appeals: A decision made or penalty imposed under *Policy 70, Student Petitions and Grievances* (other than a petition) or *Policy 71, Student Discipline* may be appealed if there is a ground. A student who believes he/she has grounds for an appeal should refer to *Policy 72, Student Appeals*,

http://www.adm.uwaterloo.ca/infosec/Policies/policy72.htm.

Students with disabilities: The *Office for Persons with Disabilities* (OPD), located in Needles Hall, Room 1132, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum.

If you require academic accommodations to lessen the impact of your disability, please register with the OPD at the beginning of each academic term.