PMATH 451/651 – Measure and Integration

Instructor: Michael Brannan
Office: MC 5447
Email: michael.brannan@uwaterloo.ca
Course Website: https://learn.uwaterloo.ca/

Lectures: MWF 9:30–10:20 in RCH 206
TAs: TBA.
Weekly office hours: TBA

Textbook:

- Online Course Notes: *PMATH 451/651 - Measure Theory*, written by K.R. Davidson. They are available at
  http://www.math.uwaterloo.ca/~krdavids/MT/PMath451Notes.pdf.
- Gerald B. Folland, *Real Analysis: Modern techniques and Their Applications*, Wiley-Interscience, 1999 (second edition).

Other resources:

- H.L. Royden, *Real Analysis*, Prentice-Hall, multiple editions.
- A.M. Bruckner, J.B. Bruckner and B.S. Thomson, *Real Analysis*, Prentice Hall, 1997. classicalrealanalysis.info/Free-Downloads.php

Course Outline

1. **Measures**: σ-algebras, measures, pre-measures, outer measures, Caratheodory's Theorem, Borel measures on $\mathbb{R}$, Lebesgue-Steiltjes measures.
2. **Integration**: measurable functions, integrals of non-negative functions, Monotone Convergence Theorem and Fatou's Lemma, integration of complex-valued functions, Dominated Convegence Theorem, modes of convergence, Egoroff's Theorem
3. **Product measure**s: Tonelli's Theorem, Fubini's Theorem, n-dimensional Lebesgue integral
4. Signed measures and differentiation: signed measures, Hahn and Jordan decompositions, total variation, Lebesgue-Radon-Nikodym Theorem and Lebesgue decomposition, complex measures, differentiation on $\mathbb{R}^n$, maximal functions, Lebesgue Differentiation Theorem, functions of bounded variation on $\mathbb{R}$, absolute continuity
5. L^p-**spaces**: basic theory, Hölder and Minkowski inequalities, dual spaces, inequalities, containment relations.
6. **Measure theory on locally compact spaces**: some topology, positive linear functionals on $C_c(X)$, Riesz Representation Theorem, regularity and approximation, Lusin's Theorem, the dual of $C_0(X)$.

PMATH 451 Grading Scheme	Homework	60 %
	Final Exam	40 %

PMATH 651 Grading Scheme	Homework	50 %
	Project	10 %
	Final Exam	40 %

Assignments: There will be approximately 5 regular assignments, due every other week. These will be posted on the course Learn site. All assignments should uploaded to the CrowdMark site for MATH 451 by the deadline (11:59PM on the due date). An assigment is considered late if it is submitted more than 24 hours past the deadline.

Late assignments will be penalized, and possibly not be graded.

Project: Graduate students taking PMATH 651 are required to complete a project, consisting of a presentation given during the pre-exam study days in April, and a written report to be submitted after the presentation.

Examinations: The final examination will take place in-person and is to be scheduled by the registrar. Do not miss the exam unless it is a medical emergency. Documentation is required and could consist of a self-declaration of illness:
`https://uwaterloo.ca/quest/help/students/how-do-i/self-declare-my-illness`

Missed final exam: An incomplete is only an option if there is a legitimate reason for missing the exam (generally a medical emergency) *and* the term work has been satisfactory.

Missed lectures: Students are expected to attend every lecture. If a lecture is missed due to an illness, or for purposes of self-isolation due to COVID-19 exposure, lecture notes will be made available to the student upon request.

Crowdmark: You sign in to Crowdmark using Learn. Crowdmark is a system for uploading questions online so that graders have easy access to your solutions, and you have easy access to their comments. Each question should be scanned or photographed separately, and uploaded into the appropriate section for that question. Students experience serious time delays when they attempt to do this close to deadline, resulting in late work. I recommend uploading each question once you think it is complete. If you change something, you can delete and add a new solution as long as you have not submitted. Be sure to save after each change. For more information, see:
https://crowdmark.com/help/categories/support-for-students/
Note: When uploading to crowdmark, you can delete and resubmit solutions to questions as many times as you like. However, once the deadline has passed, you can only upload your solution once.

The University Senate mandates that every course outline must contain the following text.

Academic Integrity: In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. Check
www.uwaterloo.ca/academicintegrity/
for more information.

Grievance: A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read *Policy 70, Student Petitions and Grievances, Section 4*,
http://www.adm.uwaterloo.ca/infosec/Policies/policy70.htm.
When in doubt please be certain to contact the department's administrative assistant who will provide further assistance.

Discipline: A student is expected to know what constitutes academic integrity to avoid committing academic offenses and to take responsibility for his/her actions. A student who is unsure whether an action constitutes an offense, or who needs help in learning how to avoid offenses (e.g., plagiarism, cheating) or about "rules" for group work/collaboration should seek guidance from the course professor, academic advisor, or the undergraduate associate dean. For information on categories of offenses and types of penalties, students should refer to *Policy 71, Student Discipline*,

http://www.adm.uwaterloo.ca/infosec/Policies/policy71.htm.
For typical penalties check *Guidelines for the Assessment of Penalties*,
http://www.adm.uwaterloo.ca/infosec/guidelines/penaltyguidelines.htm.

Appeals: A decision made or penalty imposed under *Policy 70, Student Petitions and Grievances* (other than a petition) or *Policy 71, Student Discipline* may be appealed if there is a ground. A student who believes he/she has grounds for an appeal should refer to *Policy 72, Student Appeals*, http://www.adm.uwaterloo.ca/infosec/Policies/policy72.htm.

Students with disabilities: The *Office for Persons with Disabilities* (OPD), located in Needles Hall, Room 1132, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with the OPD at the beginning of each academic term.