PMath 940 Fall 2022

Diophantine Approximation

This course will be an introduction to the study of rational approximation to real numbers. We shall first discuss Dirichlet's theorem, Hurwitz's theorem and the basic properties of continued fractions and their connection with Pell's equation $x^2-dy^2 =1$ with d a positive squarefree integer. Next we shall consider the problem of how well most real numbers (all except a set of Lebesgue measure zero) can be approximated by rationals and we shall discuss the distribution of their partial quotients. We shall contrast the typical behaviour with that for e and related numbers. We shall then turn our attention to the approximation of algebraic numbers by rationals and to the study of Diophantine equations. This subject has a wealth of deep and significant results. For instance we shall prove Roth's theorem for which he won a Fields Medal : "Let a be a real algebraic number. Then for every positive real number z there are only finitely many distinct rationals p/q with q positive and with

$$| a – p/q | < 1/q^{(2+z)}$$."

Schmidt generalized Roth's theorem to simultaneous approximation and we shall investigate some consequences of his result.

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