PMATH 940: Topics in Number Theory: Diophantine Equations & Diophantine Inequalities (C. Stewart)

In 1934 Gelfond and Schneider independently proved that if α is algebraic and different from 0 and 1 and β is algebraic and irrational then α^β is transcendental. This solved Hilbert's seventh problem. In 1967 Baker proved that if $\alpha_1, \ldots, \alpha_n$ are algebraic numbers and different from 0 and 1 and $\beta_1, \ldots, \beta_n$ are algebraic numbers for which 1, $\beta_1, \ldots, \beta_n$ are linearly independent over the rationals then $\alpha_1^{\beta_1} \ldots \alpha_n^{\beta_n}$ is transcendental. It follows from Baker's proof, for which he was awarded a Fields Medal, that a number of Diophantine equations can be solved effectively. In addition a number of previously intractable Diophantine inequalities can be resolved. Applications of Baker's estimates for linear forms in the logarithms of algebraic numbers continue to be found and in this course we shall treat some of them to show the power of the ideas behind them. For instance, we shall give effective bounds for solutions of the Thue equation and discuss Tijdeman's results on the gaps between integers composed of a fixed set of primes.

References: Exponential Diophantine equations, T.N.Shorey and R. Tijdeman, Cambridge University Press, 1986; Linear Forms in Logarithms and Applications, Y. Bugeaud, EMS IRMA Lectures in Mathematics and Theoretical Physics, Volume 28, 2018.

Textbook: no text required

Day and Time: MWF 11:30- 12:20 pm

Where: MC 5479

First class: Wednesday, September 4, 2019