RATIONAL HOMOTOPY THEORY IN GEOMETRY

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The algebra of differential forms on a smooth manifold contains more than enough information to uniquely reconstruct the topological space underlying the smooth manifold. Though it is itself unwieldy, rational homotopy theory provides a method of creating computable models that extract the homotopy theoretic information of the space modulo torsion. For example, these models - which are free algebras equipped with a differential - compute the cohomology of the manifold, and its generators correspond to the homotopy groups modulo torsion.

We will cover the basics of rational homotopy and explore various geometric applications, which will involve performing concrete calculations involving characteristic classes and homological algebra. Geometric applications can include: studying the problem of which Riemannian manifolds admit infinitely many essentially distinct geodesics; finding strong topological restrictions to admitting complex structures that satisfy the ddbar-lemma or something close to it; studying which homotopy types are realized by almost complex manifolds; and more.