Geometry Summer School 2016

On the volume spectrum in hyperbolic geometry

Luca Di Cerbo, International Centre for Theoretical Physics (ICTP) (Mathematics Section), Trieste, Italy

Wednesday, June 29 and Thursday, June 30, 2016
1:30 p.m.- 3:00 p.m.
MC 5501

Lecture 1: Wednesday June 29, 1:30 p.m. - 3:00 p.m., MC 5501

The study of volumes in hyperbolic geometry, real & complex, is a classical topic in differential geometry which is beautifully interconnected with many other parts of mathematics such as geometric topology, algebraic geometry, group theory and even computational mathematics. In this first lecture, I will introduce this fascinating subject through the study of volumes of hyperbolic Riemann surfaces via the classical Gauss-Bonnet theorem. Then, I will give an overview of some modern works on hyperbolic 3-manifolds due to Thurston and his school. Finally, I will recall the necessary background for the study of volumes in higher dimensional complex hyperbolic geometry such as the Chern-Gauss-Bonnet theorem. The last part of this lecture is preparatory for the second lecture, which will be exclusively on complex hyperbolic geometry.

Lecture 2: Thursday June 30, 1:30 p.m. - 3:00 p.m., MC 5501

In this lecture, I will focus on the study of complex hyperbolic surfaces and their volume spectrum. Recall that a complex hyperbolic surface is a complete finite volume complex surface (a manifold of real dimension four) which admits a metric of negative constant holomorphic sectional curvature. In the compact case, a problem which has attracted considerable attention is the classification of all complex hyperbolic surfaces with minimal volume. This intriguing problem, originally proposed by Mumford in the 70's, has a long and rich history which I will briefly recall. The analogous problem in the non-compact finite volume case has an interesting history as well. In fact, Hirzebruch in the 80's constructed the first example of a non-compact complex hyperbolic surface with minimal volume. After reviewing Hirzebruch's example, I will construct a new example, recently discovered in collaboration with M. Stover, and address the classification problem of all such surfaces. Finally, I will explicitly construct a tower of examples which saturates the whole admissible volume spectrum of complex hyperbolic surfaces. This is again joint with M. Stover.

Registration for this event is required - http://uwaterloo.ca/pure-mathematics/geometry-summer-school

For more information contact Benoit Charbonneau - benoit.charbonneau@uwaterloo.ca