# PMATH 445/745: Representation Theory of Finite Groups

The course introduces students to the representation theory of finite groups. It begins with a discussion of the elementary properties of linear representations of finite groups, subrepresentations, irreducibility, and various ways of combining two representations to get another: direct sums, tensor (Kronecker) products, and symmetric squares.

Next comes character theory, including the statement and proof of Schur's Lemma, the orthogonality relations, and the canonical decomposition of a representation. Together with the notion of an induced representation, the course now has the tools to explore a variety of examples.

The final part of the representation theory studies induced representations in greater detail, applying the tools developed so far to prove the Frobenius reciprocity theorem and Mackey's criterion for irreducibility. Time permitting, we apply the above theorems to investigate further particular examples of induced representations.

The course then turns to explore the connections between the representations of $G$ and modules over the group ring $\mathbb{C}[G]$. Properties of the group ring in particular, and semisimple rings more generally, are explored; including the fact that every semisimple ring is a product of matrix algebras over division rings (the Artin-Wedderburn theorem). Applications of the structure of $\mathbb{C}[G]$ to the representation theory of $G$ are discussed.

## References


[1] J.-P. Serre, "Linear Representation of finite groups," Springer-Verlag, New York, 1977.