Course Information

PMath 451/651 (Measure and Integration), Winter Term 2020

Room and time: M W F 2:30–3:20 pm, in RCH 105

Instructor: Alexandru Nica (anica@uwaterloo.ca)

Course description. This course provides a basic introduction to Lebesgue's theory of measure and integration. We will be primarily concerned with a framework where (instead of merely considering the Lebesgue measure on $\mathbb{R}$) we look at a positive measure μ on a metric space X. We will spend a bit of time to understand how such measures μ can be constructed, and why using this more general framework is useful.

We will then do a concise overview of Lebesgue's method for integrating measurable functions, upgraded to the (X, μ) framework. The main ideas for constructing the integral are pretty much the same as in the special case of Lebesgue integration on $\mathbb{R}$, and same is true about the statements and proofs of the very important convergence theorems of Lebesgue (the "monotone" and the "dominated" convergence theorem).

The remaining part of the course will be devoted to the following three topics.

• Absolute continuity, the Radon-Nikodym theorem and the Lebesgue decomposition theorem. Here we will also discuss some basic applications of Radon-Nikodym (for instance to duality for L^p-spaces).

• Product measures and the theorem of Fubini concerning iterated integrals.

• Radon integrals and the Riesz representation theorem. Here we will focus for simplicity on the case when X is a compact metric space. It is very useful that finite positive measures on X can be defined by prescribing how they integrate continuous functions. We will examine how this goes, in the form of a theorem of Riesz which identifies the dual of $C(X)$ as a space of signed measures on X.

Textbook. "Measure and Integration" by H. Bercovici, A. Brown and C. Pearcy, Springer 2016 (available online from the University of Waterloo Library).

Homework assignments. There will be weekly homework assignments, posted on the Learn web-site of the course. The homework will be submitted and graded on Crowdmark. The lowest homework score will be dropped from the grade calculation.

Please be aware that the presentation style and the clarity of your homework solutions is important, and will be factored into the grading of the assignments.

A homework assignment may occasionally include definitions and facts that are related to the questions on the assignment. Please be aware that such definitions and facts are an intrinsic part of the course, and may be tested on the exam.

Course grade. The weights for PMath 451 students are:
25% homework assignments; 10% mini-project; 65% final exam.

The weights for PMath 651 students are:
25% homework assignments; 20% project and presentation; 55% final exam.

Academic Integrity: In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. [Check www.uwaterloo.ca/ academicintegrity/ for more information.]

Grievance: A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70, Student Petitions and Grievances, Section 4, http://www.adm.uwaterloo.ca/infosec/Policies/policy70.htm. When in doubt please be certain to contact the department's administrative assistant who will provide further assistance.

Discipline: A student is expected to know what constitutes academic integrity to avoid committing academic offenses and to take responsibility for his/her actions. A student who is unsure whether an action constitutes an offense, or who needs help in learning how to avoid offenses (e.g., plagiarism, cheating) or about rules for group work/collaboration should seek guidance from the course professor, academic advisor, or the undergraduate associate dean. For information on categories of offenses and types of penalties, students should refer to Policy 71, Student Discipline, http://www.adm.uwaterloo.ca/ infosec/Policies/policy71.htm. For typical penalties check Guidelines for the Assessment of Penalties, http://www.adm.uwaterloo.ca/infosec/guidelines/penaltyguidelines.htm.

Appeals: A decision made or penalty imposed under Policy 70, Student Petitions and Grievances (other than a petition) or Policy 71, Student Discipline may be appealed if there is a ground. A student who believes he/she has a ground for an appeal should refer to Policy 72, Student Appeals, http://www.adm.uwaterloo.ca/infosec/Policies/policy72.htm.

Note for students with disabilities: AccessAbility Services (AAS), located in Needles Hall, Room 1132, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with AAS at the beginning of each academic term.

Use of laptops and mobile devices: In general, you should not photograph, record, or videotape lectures without the instructor's permission. Also, you may only use an electronic device during a lecture for course-related purposes (e.g. for note taking, if you are a fast typist).

Intellectual Property: Students should be aware that this course contains the intellectual property of their instructor, TA, and/or the University of Waterloo. It is necessary to ask the instructor, TA and/or the University of Waterloo for permission before uploading and sharing the intellectual property of others online. See policy 73 – Intellectual property rights (https://uwaterloo.ca/secretariat-general-counsel/policies-procedures-guidelines/policy-73-intellectual-property-rights)

UW Email Address: If you are corresponding with other members of the university community, it is expected that you use your University of Waterloo account (userid@uwaterloo.ca). See the "Official Student Email Address" link at uwaterloo.ca/email.