PMath 950   **PRELIMINARY Course Outline**

Topics in Analysis: problems concerning amenable operator algebras

Time: January 2022   **Place:** Only time will tell, and currently time is not talking.

Instructor: L.W. Marcoux   MC 5014

Basic Outline:

The purpose of this course is to highlight three (time permitting) famous problems in the theory of bounded linear operators and algebras of bounded linear operators acting on complex Hilbert spaces. The study of any one of these problems could easily fill a semester course by itself. Thus this course will provide an overview of the material, as opposed to a thorough and exhaustive compilation of proofs of each result we discuss. Be forewarned that I will highlight aspects of these problems which are of specific interest to me.

KADISON'S SIMILARITY PROBLEM
- Background. Completely bounded and completely positive maps
- Background. Amenability/nuclearity of operator algebras and C^*-algebras
- Positive results
- Equivalent formulations
- Open questions

REDUCTIVITY AND ORTHOGONAL REDUCTIVITY IN HILBERT SPACE
- Background. The reductive algebra problem.
- Background. The strongly reductive algebra problem.
- Background. Gifford's total reduction property
- Positive results and an inverse to Kadison's Similarity Problem
- Open questions

Time permitting, we shall also spend a couple of weeks on the following topic:

(ALMOST) INVARIANT SUBSPACES
- Background. The invariant subspace problem.
- Background. The almost invariant subspace problem.
- Positive results
- Open questions

Grading: Grades will be based upon assignments and a final presentation.

Reference materials: I will be making my notes available online, and the bibliography for these notes will serve as a reference list for the course. For a student trying to decide what kind of material will be covered and whether they might be interested in taking this course, I would recommend they consult the following resources.

- Brown, N. and Ozawa, N., *C^*-algebras and finite-dimensional approximations*
- Gifford, J.E., *Operator algebras with a reduction property - PhD thesis*
- Paulsen, V., *Completely bounded maps and dilations*
- Pisier, G., *Similarity problems and completely bounded maps*
- Popov, A. and Tcaciuc, A., *Every operator has almost-invariant subspaces*