

PMATH 446/646: Introduction to Commutative Algebra

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This course primarily targets Pure Mathematics honours students that wish to further their knowledge in algebra. It is of particular importance to students interested in algebraic geometry and algebraic number theory.

The first part of this course is a careful introduction to the theory of modules over commutative unital rings. The relevant ring and ideal theory are reviewed as needed. The following foundational notations are discussed: submodule, quotient, sum and product, finitely generated module, exact sequence tensor product, flatness, R -algebra, direct limit, localization, chain conditions. The classification of finitely generated modules over principal ideal domains is proved.

The second part of the course focuses on integral extensions and integrally closed domains. The main results here are the going up and going down theorems. Noether normalisation and Hilbert Nullstellensatz are also done in this section. The final core part of the course is the existence and uniqueness of primary decomposition for ideals in Noetherian rings. If the time permits, we will pursue additional topics such as Dedekind domains, discrete valuation rings, and dimension theory.

The textbook is *Introduction To Commutative Algebra, Student Economy Edition*, by Michael Atiyah.

References

- [1] Michael Atiyah, "Introduction To Commutative Algebra, Student Economy Edition," Westview Press; 1 edition (December 8, 2015).
- [2] David Eisenbud, "Commutative Algebra: with a View Toward Algebraic Geometry," GTM 150, Springer-Verlag, 1995.