PMATH 465/665: Smooth Manifolds
FALL 2020

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- **Office Hours:** See note [2] below

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Course Lectures: See note [1] below

NOTE: This course is being offered *online only* this term.

[1] Lectures will be posted to LEARN as short videos of about 15 to 20 minutes each. There will be approximately three hours of lectures posted every week. Lectures will be posted a few days in advance of when they would normally have been given in person during the term. I will *not* post the entire course at the start of the term for two reasons: First, I will not have it ready. I will be preparing/recording these lectures week by week as I would in a normal term. Second, it is better for your to try to learn and digest the material *slowly*. It is not possible to "binge" this material. Posting lectures on a regular basis is in *your best interest*.

[2] There *may be* a weekly in-person "tutorial" that will function essentially as office hours in a large classroom so we can all maintain physical distance. This is still tentative and subject to approval by the university. Regardless, "office hours" will also be handled using **Piazza**. You can expect a response from me within two hours most of the time, if your question is posted between 8am and 10pm. Questions posted in the middle of the night will be answered the following morning. One-on-one virtual appointments may also be possible.

Course description: Point-set topology; smooth manifolds, smooth maps, and tangent vectors; the tangent and cotangent bundles; vector fields, tensor fields, and differential forms; Stokes's Theorem; integral curves, Lie derivatives, the Frobenius Theorem; de Rham cohomology.

Prerequisites: For undergraduates registered in PMATH 465, the strict prerequisite is PMATH 365.

Textbook: There is no required textbook. However, there are many excellent books that cover this material. Here are two useful books, which will both be on reserve at the Davis library:

- Introduction to Smooth Manifolds; Second Edition; by Lee (Springer GTM)
- An Introduction to Differentiable Manifolds and Riemannian Geometry; by Boothby (Academic Press)

I will not prepare my lectures by following any particular book exactly. This is a good thing for you, since by viewing the lectures and by also reading various books you will have multiple points of view on the topics, which will make it easier to understand. The best way to learn is to read books, view the lectures, and then read books again. (And then read books a third or fourth time if necessary.)

Marking scheme: There will be six assignments (*one due every two weeks*), and also a final assessment during the final exam period. Assignments will be lengthy and often technical. *Do not leave them for the last minute. Start them right away.* Graduate students are also required to write a 10–15 page typewritten paper on a topic related to the course (approved by the instructor), and to give a 45 minute online oral presentation on the topic of the paper. The presentations will all be held after the final lecture, but before the final exam period.

You are encouraged to work together with your classmates on the assignments, but you must write up and turn in your own solutions to the problems. The assignments are an integral part of your evaluation in this course and I encourage everyone to take them very seriously. I will not be sympathetic to requests for leniency after the final assessment if you have not done the assignments. Late assignments will *not* be accepted.

For the final assessment, looking at books or notes, whether online or hard copy, is permitted, but you are *not* allowed to work together on the final assessment, *nor are you permitted to ask questions on online forums.*

You cannot pass this course unless you obtain a grade of at least 50% on the final assessment.

Your course mark will be determined as follows:

- For undergraduate students registered in PMATH 465:
  - Assignments: 78% FINAL EXAM: 22%
- For graduate students registered in PMATH 665:
  - Assignments: 60% FINAL EXAM: 22% Paper/Presentation: 18%

Detailed outline of course topics. (Tentative and subject to change.)

[1] point-set topology, compactness, connectedness; quotient spaces; topological manifolds

[2] abstract smooth manifolds, examples including spheres, projective spaces, and Lie groups

[3] smooth functions, bump functions, partitions of unity

[4] tangent spaces, tangent bundle, vector fields, Lie bracket

[5] cotangent bundle, 1-forms, tensor bundles, tensors

[6] flows, Lie derivatives, Frobenius theorem, (optional: applications to ODE)

[7] submanifolds, immersions, embeddings; submersions

[8] orientations, differential forms, integration (including partitions of unity), Stokes's theorem

[9] de Rham cohomology, computation

[10] if time permits or sprinkled throughout in examples: embedding theorems, Lie groups, homogeneous spaces.

Note: Section [8] will be covered quickly because the undergrads will have already seen a special case of it in PMATH 365. Alternatively, it may even just be assigned as independent reading for the graduate students who did not take PMATH 365, giving them a separate assignment on this material.

Academic offences

Academic Integrity: In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. Please see `http://www.uwaterloo.ca/academicintegrity/` for more information.

Grievance: A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70 - Student Petitions and Grievances, Section 4, `http://www.adm.uwaterloo.ca/infosec/Policies/policy70.htm`. When in doubt please be certain to contact the department's administrative assistant who will provide further assistance.

Discipline: A student is expected to know what constitutes academic integrity, to avoid committing academic offenses, and to take responsibility for his/her actions. A student who is unsure whether an action constitutes an offense, or who needs help in learning how to avoid offenses (e.g., plagiarism, cheating) or about rules for group work/collaboration should seek guidance from the course professor, academic advisor, or the Undergraduate Associate Dean. For information on categories of offenses and types of penalties, students should refer to Policy 71, Student Discipline, `http://www.adm.uwaterloo.ca/infosec/Policies/policy71.htm`. For typical penalties check Guidelines for the Assessment of Penalties, `http://www.adm.uwaterloo.ca/infosec/guidelines/penaltyguidelines.htm`.

Avoiding Academic Offenses: Most students are unaware of the line between acceptable and unacceptable academic behaviour, especially when discussing assignments with classmates and using the work of other students. For information on commonly misunderstood academic offenses and how to avoid them, students should refer to the Faculty of Mathematics Cheating and Student Academic Discipline Policy, `http://www.math.uwaterloo.ca/navigation/Current/cheating_policy.shtml`

Appeals: A student may appeal the finding and/or penalty in a decision made under Policy 70 - Student Petitions and Grievances (other than regarding a petition) or Policy 71 - Student Discipline if a ground for an appeal can be established. Read Policy 72 - Student Appeals, `http://www.adm.uwaterloo.ca/infosec/Policies/policy72.htm`

Note for students with disabilities

The AccessAbility Services (AS) Office, located in Needles Hall, Room 1132, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with the AS Office at the beginning of each academic term.
