PMATH 465/665: Smooth Manifolds
FALL 2021

- **Instructor:** Spiro Karigiannis
- **Telephone:** 519-888-4567 ext 42810
- **Office:** MC 5326
- **Email:** karigiannis@uwaterloo.ca
- **Office Hours:** Tue/Thu 2:00pm–3:00pm or by appointment
- **Course Lectures:** Mon/Wed/Fri 9:30am–10:20am in MC 2017

NOTE: This course is being offered in person on campus this term, *not online*. Some asynchronous resources may be made available, but if you are not expecting to be attending the course lectures in person, please contact me, because this is definitely not recommended.

Course description: Point-set topology; smooth manifolds, smooth maps, and tangent vectors; the tangent and cotangent bundles; vector fields, tensor fields, and differential forms; Stokes's Theorem; integral curves, Lie derivatives, the Frobenius Theorem; de Rham cohomology.

Prerequisites: For undergraduates registered in PMATH 465, the strict prerequisite is PMATH 365.

Textbook: There is no required textbook. However, there are many excellent books that cover this material. Here are two useful books, which will both be on reserve at the Davis library:

- Introduction to Smooth Manifolds; Second Edition; by Lee (Springer GTM)
- An Introduction to Differentiable Manifolds and Riemannian Geometry; by Boothby (Academic Press)

I will not prepare my lectures by following any particular book exactly. This is a good thing for you, since by viewing the lectures and by also reading various books you will have multiple points of view on the topics, which will make it easier to understand. The best way to learn is to read books, view the lectures, and then read books again. (And then read books a third or fourth time if necessary.)

Marking scheme: There will be six assignments (*one due every two weeks*), and a final exam. Assignments will be lengthy and often technical. *Do not leave them for the last minute. Start them right away.* Graduate students are also required to write a 10–15 page typewritten paper on a topic related to the course (approved by the instructor), and to give a 45 minute oral presentation on the topic of the paper. The presentations will all be held after the final lecture, but before the final exam period.

The assignments are constructed very carefully to clarify and deepen your understanding of the concepts. I believe that at least half of your learning comes from working through the assignment problems in detail. I encourage you to work together with your classmates on the assignments, but you must write up and turn in your own solutions to the problems. The assignments are an integral part of your evaluation in this course and I advise everyone to take them very seriously. Late assignments will *not* be accepted.

The final exam will be a traditional closed book 2.5 hour final exam held during the final exam period.

You cannot pass this course unless you obtain a grade of at least 50% on the final exam.

Your course mark will be determined as follows:

- For undergraduate students registered in PMATH 465:
  - Assignments: 66%   Final Exam: 34%

- For graduate students registered in PMATH 665:
  - Assignments: 54%   Final Exam: 32%   Paper/Presentation: 14%

Detailed outline of course topics. (Tentative and subject to change.)

[1] point-set topology, compactness, connectedness; quotient spaces; topological manifolds

[2] abstract smooth manifolds, examples including spheres, projective spaces, and Lie groups

[3] smooth functions, bump functions, partitions of unity

[4] tangent spaces, tangent bundle, vector fields, Lie bracket

[5] cotangent bundle, 1-forms, tensor bundles, tensors

[6] flows, Lie derivatives, Frobenius theorem, (if time permits: applications to differential equations)

[7] submanifolds, immersions, embeddings; submersions

[8] orientations, differential forms, integration (including partitions of unity), Stokes's theorem

[9] de Rham cohomology, computation

[10] sprinkled in throughout assignments: applications to Lie groups, symplectic forms, Riemannian metrics

Pandemic contingency planning:

Cancelled lectures: In the event that there is a cancellation of in-person classes due to the pandemic (whether for short-term or long-term), then pre-recorded lectures from the Fall 2020 offering of the course will be uploaded. The course instructor taught the course online last year, so these recordings should be similar to (but probably not exactly the same, particularly the notation) as the in-person lectures in Fall 2021. If last year's recordings need to be posted, then the instructor will detail any differences (especially in notation) from this year's offering, and additional online office hours will be offered during the period of disruption to in-person lectures. Similarly, if any particular students needs to self-isolate during the term, they can be given access to last year's recordings.

Final exam: If the in-person final exam needs to be cancelled due to the pandemic, it will be replaced with a fully online "take-home exam" worth the same amount as the final exam.

Academic offences

Academic Integrity: In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. Please see http://www.uwaterloo.ca/academicintegrity/ for more information.

Grievance: A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70 - Student Petitions and Grievances, Section 4, http://www.adm.uwaterloo.ca/infosec/Policies/policy70.htm. When in doubt please be certain to contact the department's administrative assistant who will provide further assistance.

Discipline: A student is expected to know what constitutes academic integrity, to avoid committing academic offenses, and to take responsibility for his/her actions. A student who is unsure whether an action constitutes an offense, or who needs help in learning how to avoid offenses (e.g., plagiarism, cheating) or about ÒrulesÓ for group work/collaboration should seek guidance from the course professor, academic advisor, or the Undergraduate Associate Dean. For information on categories of offenses and types of penalties, students should refer to Policy 71, Student Discipline, http://www.adm.uwaterloo.ca/infosec/Policies/policy71.htm. For typical penalties check Guidelines for the Assessment of Penalties, http://www.adm.uwaterloo.ca/infosec/guidelines/penaltyguidelines.htm.

Avoiding Academic Offenses: Most students are unaware of the line between acceptable and unacceptable academic behaviour, especially when discussing assignments with classmates and using the work of other students. For information on commonly misunderstood academic offenses and how to avoid them, students should refer to the Faculty of Mathematics Cheating and Student Academic Discipline Policy, http://www.math.uwaterloo.ca/navigation/Current/cheating_policy.shtml

Appeals: A student may appeal the finding and/or penalty in a decision made under Policy 70 - Student Petitions and Grievances (other than regarding a petition) or Policy 71 - Student Discipline if a ground for an appeal can be established. Read Policy 72 - Student Appeals, http://www.adm.uwaterloo.ca/infosec/Policies/policy72.htm

Note for students with disabilities

The AccessAbility Services (AS) Office, located in Needles Hall, Room 1132, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with the AS Office at the beginning of each academic term.