Course Information for PMath 651 (Measure and Integration) in Winter Term 2023

Room and time.
Mondays and Wednesdays 4 to 5:20 pm, in EV1 Room 132.

Instructor.
Alexandru Nica (Professor, Pure Mathematics Department), anica@uwaterloo.ca

Course description.
This course provides a basic introduction to Lebesgue's theory of measure and integration. The primary framework we use is the one of a *measure space* $(X, \mathcal{M}, \mu)$. We will start the course by looking at how such spaces can be constructed, and why using this general framework is useful. We will then do a concise overview of Lebesgue's method for integrating measurable functions, upgraded to the $(X, \mathcal{M}, \mu)$ framework. The main ideas for constructing the integral are pretty much the same as in the special case of Lebesgue integration on $\mathbb{R}$ (so it will be helpful if you had some previous exposure to that special case); same is true about the statements and proofs of the very important convergence theorems of Lebesgue, the "monotone" and the "dominated" convergence theorem.

The remaining part of the course will be devoted to the following three topics.

- Product measures and the theorem of Fubini concerning iterated integrals.
- Absolute continuity, the Radon-Nikodym theorem and some of its applications; this includes some discussion about L^p-spaces, and about signed measures.
- Radon integrals. Here we will discuss the Riesz representation theorem for a compact metric space X. This theorem identifies the bounded linear functionals on the space $C(X)$ as signed measures on X.

Lectures.
I will attempt to make the sequence of lectures clear and self-contained, so that your notes from class can be themselves used as primary reference, when you work on the homework assignments or you prepare for the final exam.

Homework assignments.
There will be weekly homework assignments, posted on the Learn web-site of the course. The homework will be submitted and graded on Crowdmark. The lowest homework score will be dropped from the grade calculation.

Please be aware that the presentation style and the clarity of your homework solutions is important, and will be factored into the grading of the assignments.

A homework assignment may occasionally include definitions and facts that are related to the questions on the assignment. Please be aware that such definitions and facts are an intrinsic part of the course, and may be tested on the exam.

Project.
One of the elements considered in the course-grade is a "project" – that is, a little *paper* you will have to write on a topic related to what we do in the course. It is of course recommendable that you pick, if at all possible, a topic which is at the same of interest for the direction you are planning to pursue as a MMath student (if in doubt whether a topic

is suitable, please don't hesitate to consult with me about it). There are also many possible project topics which can be selected out of the references indicated below.

The projects will be due at the end of the term, but you will have to submit the *title and abstract* of your paper with a few weeks in advance of that.

References.
There are many books on measure and integration. Some "classics" in this area are [1] and [2], for a more recent treatment you could look into [3] (which is available online on the UW Library web-site, via SpringerLink). If you want to do a project going in a probability direction, you could look into [4], [5], or perhaps (more ambitious) into [6]. A very nice survey paper going in the direction of "geometric measure theory" and which can serve as reference for a project is [7].

[1] W. Rudin. *Real and Complex Analysis, 3rd edition.* published by McGraw-Hill, 1987.

[2] H.L. Royden. *Real Analysis, 3rd edition*, published by Prentice-Hall, 1988.

[3] H. Bercovici, A. Brown, C. Pearcy. *Measure and Integration*, Springer, 2016.

[4] P. Billingsley. *Probability and measure*, anniversary edition, John Wiley, 2012.

[5] R.M. Dudley. *Real analysis and probability*, 2nd edition, Cambridge Univ. Press, 2003.

[6] J-F LeGall. *Brownian motion, martingales, and stochastic calculus*, Springer, 2016.

[7] R.J. Gardner. The Brunn-Minkowski inequality, *Bulletin of the American Mathematical Society* 39 (2002), 355-405.

Weights for computing the PMath 651 course-grade.

- → Homework assignments: 35%.
- → Project (on a topic agreed upon with the instructor): 20%.
- → Final Exam (held together with the one of PMath 451): 45%.

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Academic Integrity: In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. [Check https://uwaterloo.ca/academic-integrity/ for more information.]

Grievance: A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70, Student Petitions and Grievances, Section 4,
https://uwaterloo.ca/secretariat/policies-procedures-guidelines/policy-70
When in doubt please be certain to contact the department's administrative assistant who will provide further assistance.

Discipline: A student is expected to know what constitutes academic integrity to avoid committing academic offenses and to take responsibility for his/her actions. A student who is unsure whether an action constitutes an offense, or who needs help in learning how to avoid offenses (e.g., plagiarism, cheating) or about rules for group work/collaboration should seek guidance from the course professor, academic advisor, or the undergraduate

associate dean. For information on categories of offenses and types of penalties, students should refer to Policy 71, Student Discipline,
https://uwaterloo.ca/secretariat/policies-procedures-guidelines/policy-71

Appeals: A decision made or penalty imposed under Policy 70, Student Petitions and Grievances (other than a petition) or Policy 71, Student Discipline may be appealed if there is a ground. A student who believes he/she has a ground for an appeal should refer to Policy 72, Student Appeals,
https://uwaterloo.ca/secretariat/policies-procedures-guidelines/policy-72

Note for students with disabilities: AccessAbility Services (AAS),
https://uwaterloo.ca/accessability-services/, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with AAS at the beginning of each academic term.

Intellectual Property: Students should be aware that this course contains the intellectual property of their instructor, TA, and/or the University of Waterloo. It is necessary to ask the instructor, TA and/or the University of Waterloo for permission before uploading and sharing the intellectual property of others online. See policy 73 – Intellectual property rights (https://uwaterloo.ca/secretariat/policies-procedures-guidelines/policies/policy-73-intellectual-property-rights)

UW Email Address: If you are corresponding with other members of the university community, it is expected that you use your University of Waterloo account (userid@uwaterloo.ca). See the "Official Student Email Address" link at uwaterloo.ca/email.