PMATH 868: Connections and Riemannian Geometry
WINTER 2021

- **Instructor:** Spiro Karigiannis
- **Telephone:** 519-888-4567 ext 32810
- **Office Hours:** See note below.

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Course Lectures: TBA

NOTE: This course is being offered in-person on campus, *not online*. If the university allows me to hold in-person office hours, I will. This may have to be in the form of a weekly in-person "tutorial" that will function essentially as office hours in a large classroom so we can all maintain physical distance. Regardless, "office hours" will also be handled using **Piazza**. You can expect a response from me within two hours most of the time, if your question is posted between 8am and 10pm. Questions posted in the middle of the night will be answered the following morning. One-on-one virtual appointments may also be possible.

Course description: Review of smooth manifolds. Vector bundles. Connections and curvature, holonomy, characteristic classes. Connections on tangent bundle: torsion, geodesics, exponential map. Riemannian geometry: Levi-Civita connection, Riemannian geodesics, Hopf-Rinow Theorem. Additional topics if time permits.

Prerequisites: PMATH 665 is an essential prerequisite.

Textbook: There is no required textbook. I plan to write course notes as we go along during the term.

Detailed outline of course topics. (Tentative and subject to change.)

[1] quick review of smooth manifolds, tangent bundle, tensor bundles, differential forms, to fix notation

[2] vector bundles over smooth manifolds; constructions of vector bundles; vector-valued forms

[3] connections on vector bundles, exterior covariant derivative, curvature, Bianchi identity

[4] holonomy of connections on vector bundles, flat connections and relation to the Frobenius theorem

[5] theory of characteristic classes of vector bundles

[6] the tangent bundles case: affine connections, torsion, geodesics, the exponential map

[7] Riemannian metrics, Levi-Civita connection, Riemannian geodesics, Hopf-Rinow theorem

[8] possible additional topics if time permits: Jacobi fields, isometric immersions, metrics on Lie groups

Notes on syllabus:

(a) This course is a hybrid of $\frac{1}{3}$ of a course on Riemannian geometry, *following* $\frac{2}{3}$ of a course on general connections, curvature, holonomy, and topology of vector bundles. This approach has the advantage that it sets up a lot of general geometric notions that are useful to all the geometry graduate students before specializing to the standard material on Riemannian geometry.

(b) There are many places in the above syllabus where the 665 material is essential: Frobenius theorem for the study of flatness, de Rham cohomology for characteristic classes, flows for geodesics, etc.

(c) Any undergraduates wanting to take 868 should have taken *both* 465 and 351. We require ideas from metric space theory for the discussion of the Hopf-Rinow theorem. This should not cause problems. Undergraduate students who get to 868 will have certainly had 351.

Marking scheme: There will be six assignments, *one due every two weeks*. The assignments will be lengthy and often technical. *Do not leave them for the last minute. Start them right away.* Depending on the total enrolment, the sixth assignment might be replaced with a 10–15 page typewritten paper on a topic related to the course (approved by the instructor), and a 45 minute oral presentation on the topic of the paper. The presentations will all be held after the final lecture.

You are encouraged to work together with your classmates on the assignment problems, but you must write up and turn in your own solutions to the problems.

Your course mark will be determined by one of the following schemes, which will be chosen by reading week:

- Assignments: 100% (six assignments at 16.67% each)
- Assignments: 80% (five assignments at 16% each)   Paper/Presentation: 20%

Academic offences

Academic Integrity: In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. Please see `http://www.uwaterloo.ca/academicintegrity/` for more information.

Grievance: A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70 - Student Petitions and Grievances, Section 4, `http://www.adm.uwaterloo.ca/infosec/Policies/policy70.htm`. When in doubt please be certain to contact the department's administrative assistant who will provide further assistance.

Discipline: A student is expected to know what constitutes academic integrity, to avoid committing academic offenses, and to take responsibility for his/her actions. A student who is unsure whether an action constitutes an offense, or who needs help in learning how to avoid offenses (e.g., plagiarism, cheating) or about ÒrulesÓ for group work/collaboration should seek guidance from the course professor, academic advisor, or the Undergraduate Associate Dean. For information on categories of offenses and types of penalties, students should refer to Policy 71, Student Discipline, `http://www.adm.uwaterloo.ca/infosec/Policies/policy71.htm`. For typical penalties check Guidelines for the Assessment of Penalties, `http://www.adm.uwaterloo.ca/infosec/guidelines/penaltyguidelines.htm`.

Avoiding Academic Offenses: Most students are unaware of the line between acceptable and unacceptable academic behaviour, especially when discussing assignments with classmates and using the work of other students. For information on commonly misunderstood academic offenses and how to avoid them, students should refer to the Faculty of Mathematics Cheating and Student Academic Discipline Policy, `http://www.math.uwaterloo.ca/navigation/Current/cheating_policy.shtml`

Appeals: A student may appeal the finding and/or penalty in a decision made under Policy 70 - Student Petitions and Grievances (other than regarding a petition) or Policy 71 - Student Discipline if a ground for an appeal can be established. Read Policy 72 - Student Appeals, `http://www.adm.uwaterloo.ca/infosec/Policies/policy72.htm`

Note for students with disabilities

The AccessAbility Services (AS) Office, located in Needles Hall, Room 1132, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with the AS Office at the beginning of each academic term.