PMATH 868: Connections and Riemannian Geometry
WINTER 2023

- **Instructor:** Spiro Karigiannis — **Email:** karigiannis@uwaterloo.ca
- **Telephone:** 519-888-4567 ext 42810 — **Office Hours:** TBD
- **Office:** MC 5326 — **Course Lectures:** TBD

Course description: Review of smooth manifolds. Vector bundles. Connections and curvature, holonomy, characteristic classes. Connections on tangent bundle: torsion, geodesics, exponential map. Riemannian geometry: Levi-Civita connection, Riemannian geodesics, Hopf-Rinow Theorem. Additional topics if time permits.

Prerequisites: PMATH 665 is an essential prerequisite.

Textbook: There is no required textbook. I have written course notes for the first two thirds of the course, and hope to finish the rest of the course notes before we get to the last part of the course.

Detailed outline of course topics. (Tentative and subject to change.)

[1] quick review of smooth manifolds, tangent bundle, tensor bundles, differential forms, to fix notation

[2] vector bundles over smooth manifolds; constructions of vector bundles; vector-valued forms

[3] connections on vector bundles, exterior covariant derivative, curvature, Bianchi identity

[4] parallel transport and holonomy, the Ambrose–Singer theorem, flat connections and the fundamental group

[5] Chern–Weil theory of characteristic classes of vector bundles

[6] connections on the tangent bundles: torsion, geodesics, the exponential map

[7] Riemannian metrics, Levi-Civita connection, Riemannian geodesics, Hopf-Rinow theorem

[8] if time permits: isometric immersions, Riemannian submersions, metrics on Lie groups

Notes on syllabus:

(i) This course is a hybrid of $\frac{1}{3}$ of a course on Riemannian geometry, *following* $\frac{2}{3}$ of a course on general connections, curvature, holonomy, and topology of vector bundles. This approach has the advantage that it sets up a lot of general geometric notions that are useful to all the geometry graduate students before specializing to the standard material on Riemannian geometry.

(ii) There are many places in the above syllabus where the 665 material is essential: Frobenius theorem for the study of flatness, de Rham cohomology for characteristic classes, flows for geodesics, etc.

(iii) Any undergraduates wanting to take 868 should have taken *both* 465 and 351. We require ideas from metric space theory for the discussion of the Hopf-Rinow theorem. This should not cause problems. Undergraduate students who get to 868 will have certainly had 351.

Marking scheme: There will be six assignments, *one due every two weeks*. The assignments will be lengthy and often technical. *Do not leave them for the last minute. Start them right away.*

You are encouraged to work together with your classmates on the assignment problems, but you must write up and turn in your own solutions to the problems. Your course mark will be determined by the following scheme:

- Assignments: 100% (six assignments at 16.67% each)

NOTE: For information on academic offences and accessibility services, please see the detailed version of the course outline available at TBD