PMATH 965: Topics in Geometry and Topology
A second course in Riemannian Geometry
FALL 2022

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- **Office:** MC 5326
- **Office Hours:** Friday 10:30am–11:30am (hybrid)
- **Lecture Room:** MC 5403
- **Lecture Times:** Monday/Wednesday 10:30am–11:50am

This course is cross-listed as part of the Fields Academy Shared Graduate Courses for 2022–2023.

Course description: This is a second course in Riemannian geometry. The emphasis will be on the intimate relationship between curvature and geodesics.

Prerequisites: Students should be thoroughly familiar with smooth manifold theory, and some exposure to the basics of Riemannian geometry, including Riemannian metrics, the Levi-Civita connection, Riemann curvature, and Riemannian geodesics (as covered in PMATH 868) is helpful but not absolutely essential.

Textbook: There is no required textbook for this course. I will be following this book quite closely, however:

- M. P. do Carmo, *Riemannian geometry*, translated from the second Portuguese edition by Francis Flaherty, Mathematics: Theory & Applications, Birkhäuser Boston, Inc., Boston, MA, 1992. MR1138207

I will likely change notation from do Carmo, and I will certainly change the sign and normalization conventions for curvature to the standard ones. Other useful references are:

- S. Gallot, D. Hulin and J. Lafontaine, *Riemannian geometry*, third edition, Universitext, Springer-Verlag, Berlin, 2004. MR2088027
- J. Jost, *Riemannian geometry and geometric analysis*, seventh edition, Universitext, Springer, Cham, 2017. MR3726907
- J. M. Lee, *Introduction to Riemannian manifolds*, second edition, Graduate Texts in Mathematics, 176, Springer, Cham, 2018. MR3887684

Brief outline of course topics. (Tentative and definitely subject to change.)

[1] Review of the basics of Riemannian geometry: metrics, Levi-Civita connection, geodesics, curvature.
[2] minimizing properties of geodesics; totally normal neighbourhoods
[3] Jacobi fields and conjugate points
[4] isometric immersions and the second fundamental form
[5] completeness and the Hopf–Rinow Theorem; the Hadamard theorem; spaces of constant curvature
[6] first and second variations of energy; the Bonnet–Myers Theorem; the Synge–Weinstein Theorem
[7] the Rauch Comparison Theorem; the index lemma; focal points
[8] the Morse Index Theorem
[9] existence of closed geodesics; Preissman's Theorem
[10] cut points, the cut locus, and the injectivity radius; the Sphere Theorem.

Marking scheme

Course marks will be determined as follows.

- Assignments: 100% (five assignments, roughly every two weeks starting in third week, worth 20% each)

Please note that you are encouraged to work together with your classmates on the assignment problems, but you must write up and turn in your own solutions to the problems.

NOTE: For information on academic offences and accessibility services, please see the detailed version of the course outline available at: URL to be determined.