PMath 990 in Winter Term 2025:
Introduction to Random Matrix Theory

Day/time/room: Tu Th 1–2:20 pm, in MC 5417

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The objective of this course is to offer an introduction to random matrix theory to students who have a good background in real analysis, including measure and integration, but did not have much exposure to probability.
Here are some specifics concerning the content of the course. Our job will be to look at eigenvalues of random Hermitian $d \times d$ matrices and to study the behaviour of the said eigenvalues in the $d \to \infty$ limit. Since we want to study *random* matrices, we will need to deal with spaces of matrices that are endowed with a probability measure on their Borel sigma-algebra. Towards that end, we will examine the so-called *Wigner model* (a natural recipe for introducing a probability measure on $\{A \in \mathcal{M}_d(\mathbb{C}) \mid A = A^*\}$), and we will also examine a model where randomness is introduced by using the Haar measure of the compact group $\mathcal{U}(d)$ of $d \times d$ unitary matrices. In either model, it turns out that one can put into evidence a phenomenon called *asymptotic free independence* which arises in the $d \to \infty$ limit, and allows for neat calculations of eigenvalue distributions, e.g. in connection to the sum of two Wigner matrices with independent entries. This will allow us to dabble a bit in an area known as *free probability*, which can be viewed, in some sense, as a "$d = \infty$" sibling of random matrix theory.

Some concrete details about how the course is run. I will use a rigorous presentation style (specific to a Pure Math course) and I will attempt to make the sequence of lectures clear and self-contained, so that your in-class notes can themselves be used as reference towards absorbing the course material. When needed, I will draw on the course-notes [1], [2] indicated below, coming from recent courses on random matrices taught at other universities. The course grade will be based on: attendance, some light homework throughout the term, meant to keep you in touch with what is done in class, and an "essay-like" assignment (on a topic of your choice, but related at least loosely to random matrices) to be submitted on the last day of class.

References. [1] and [2] are notes from courses on random matrices, while [3] and [4] are research monographs that are often cited in connection to this topic.

[1] Todd Kemp, notes for course on random matrix theory taught at University of California San Diego in 2022, https://mathweb.ucsd.edu/∼tkemp/247A.Notes.pdf

[2] Roland Speicher, notes for course on random matrices taught at Saarland University in 2020, https://arxiv.org/pdf/2009.05157.

[3] G.W. Anderson, A. Guionnet, O. Zeitouni. An introduction to random matrices, Cambridge University Press, 2010. (See https://cims.nyu.edu/∼zeitouni/cupbook.pdf)

[4] J.A. Mingo, R. Speicher. Random matrices and free probability, Springer Verlag, 2017. (See https://rolandspeicher.files.wordpress.com/2019/02/mingo-speicher.pdf)