PM 965 Topics in Geometry and Topology:
Special Riemannian Structures
WINTER 2019

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- **Office Hours:** Mon/Wed/Fri 1:00pm-2:00pm

- **Course Lectures:** Mon/Wed 9:30am-10:50am, in MC 5403
- **Course Website:** UW-LEARN

Course description: We consider general affine connections on manifolds, including their torsion, curvature, and holonomy. Then we focus on the Levi-Civita connection of a Riemannian manifold, and study some basic Riemannian geometry and properties of Riemannian holonomy. Then in the final third of the course we consider a particular class of special Riemannian manifolds, those with Riemannian holonomy in $\mathrm{U}(m)$ or $\mathrm{SU}(m)$, known as Kähler and Calabi-Yau manifolds, respectively.

Prerequisites: In addition to undergraduate level complex analysis, real analysis, linear algebra, and metric space topology, the following graduate level courses are required:

- PMATH 665: *Geometry of Manifolds* is an absolutely required prerequisite.
- A course in algebraic topology (such as PMATH 667) is helpful but not strictly necessary.

Detailed list of topics

Part One: Affine connections, torsion, curvature, and holonomy

[1] Connections on the tangent bundle. Induced connections on tensor bundles.
[2] Torsion and curvature of affine connections and their geometric interpretations.
[3] Geodesics. The exponential map. Geodesic flow.
[4] Parallel transport and holonomy. Parallel tensors and holonomy reduction. Ambrose–Singer theorem.

Part Two: Riemannian geometry and Riemannian holonomy

[6] Riemannian metrics. Examples. The Levi-Civita connection.
[7] Isometries and Killing vector fields. Examples.
[8] Riemannian geodesics. Examples of geodesics. Riemannian normal coordinates.
[9] Variational characterization of Riemannian geodesics and length minimization.
[10] Geodesic completeness and metric space completeness: The Hopf–Rinow Theorem.
[11] Flatness, Riemann curvature tensor, Bianchi identities, symmetries of the Riemann curvature.
[12] Ricci and scalar curvature. Geometric interpretation of Riemann curvature.
[13] Harmonic forms. Hodge theorem for compact oriented Riemannian manifolds. Poincaré duality.
[14] The Bochner–Weitzenböck formula. Consequences.
[15] Locally reducible metrics and their holonomy. Locally symmetric metrics and their holonomy.
[16] Berger's list of holonomy groups for not locally reducible, not locally symmetric Riemannian metrics.

Part Three: Manifolds with $\mathrm{U}(m)$ or $\mathrm{SU}(m)$ structure

[18] Almost complex structures. The Newlander-Nirenberg theorem.
[19] The ∂ and $\bar{\partial}$ operators. Dolbeault cohomology of complex manifolds.
[20] Kähler manifolds. Equivalent characterizations of the Kähler condition.
[21] Curvature of Kähler metrics. The Ricci form of a Kähler manifold. Relation to the first Chern class.
[22] The Kähler identities. The Hodge decomposition for Kähler manifolds. The $\partial\bar{\partial}$-lemma.
[23] Necessary conditions for existence of Ricci-flat Kähler metric. Calabi–Yau Theorem. Sketch of proof.
[24] The Hard Lefschetz theorem. Formality of Kähler manifolds.

Textbook

There is no official textbook. I will be using many different sources for this course. Some of these are:

- Lee; *Riemannian Manifolds: An Introduction to Curvature*; Springer
- Boothby; *An Introduction to Differentiable Manifolds and Riemannian Geometry*; Academic Press
- do Carmo; *Riemannian Geometry*; Birkhäuser
- Huybrechts; *Complex Geometry*; Springer.
- Morianu; *Lectures on Kähler Geometry*; European Mathematical Society.

Marking scheme

Course marks will be determined as follows.

- Assignments: 84% (six assignments, about one every two weeks, 14% each)
- Paper/presentation (approx 10 pages, typewritten; 12% for paper and 4% for presentation)

Please note that you are encouraged to work together with your classmates on the assignment problems, but you must write up and turn in your own solutions to the problems.

Academic offenses

Academic Integrity: In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. Please see `http://www.uwaterloo.ca/academicintegrity/` for more information.

Grievance: A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70 - Student Petitions and Grievances, Section 4, `http://www.adm.uwaterloo.ca/infosec/Policies/policy70.htm`. When in doubt please be certain to contact the departments administrative assistant who will provide further assistance.

Discipline: A student is expected to know what constitutes academic integrity, to avoid committing academic offenses, and to take responsibility for his/her actions. A student who is unsure whether an action constitutes an offense, or who needs help in learning how to avoid offenses (e.g., plagiarism, cheating) or about rules for group work/collaboration should seek guidance from the course professor, academic advisor, or the Undergraduate Associate Dean. For information on categories of o?enses and types of penalties, students should refer to Policy 71, Student Discipline, `http://www.adm.uwaterloo.ca/infosec/Policies/policy71.htm`. For typical penalties check Guidelines for the Assessment of Penalties, `http://www.adm.uwaterloo.ca/infosec/guidelines/penaltyguidelines.htm`.

Avoiding Academic Offenses: Most students are unaware of the line between acceptable and unacceptable academic behaviour, especially when discussing assignments with classmates and using the work of other students. For information on commonly misunderstood academic offenses and how to avoid them, students should refer to the Faculty of Mathematics Cheating and Student Academic Discipline Policy, `http://www.math.uwaterloo.ca/navigation/Current/cheating_policy.shtml`

Appeals: A student may appeal the finding and/or penalty in a decision made under Policy 70 - Student Petitions and Grievances (other than regarding a petition) or Policy 71 - Student Discipline if a ground for an appeal can be established. Read Policy 72 - Student Appeals, `http://www.adm.uwaterloo.ca/infosec/Policies/policy72.htm`

Note for students with disabilities

The AccessAbility Services (AS) Office, located in Needles Hall, Room 1132, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with the AS Office at the beginning of each academic term.