

Math 446 and 646: Commutative Algebra

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Classroom: Does not apply

Disclaimer

Due to the nature of the Winter 2020 term, everything in this outline is subject to change. Any such change would be communicated in as far advance as is reasonable, and will be made in a manner that is as fair as possible.

Course Description

Mathematical Content

Commutative algebra is the study of rings¹ and modules over them. This course will aim to give a concise introduction to a lot of the aspects of commutative algebra, including various topics such as exact sequences, tensor products, localization, chain conditions, primary decomposition, Noether normalization, and the Nullstellensatz.

It is impossible to talk about commutative algebra without at least alluding to algebraic geometry. Many² of the examples given in this course will be geometrically motivated. Additionally, a lot of the major theorems will have geometric interpretations or become more clear when viewed through a geometric lens. This will be discussed during the course, as a way to shed light on things that may seem inscrutable when viewed through a purely algebraic lens.

Other Things

This course will be run online. I will hold lectures and office hours online and I will be most easily accessed by email. If issues arise, please feel free to contact me and I will try to work them out.

¹Throughout this course, a ring will mean a commutative ring with unity. Additionally, after this gets defined, all rings should be assumed to be noetherian unless stated otherwise

²realistically, almost all

Materials

- This course will be using David Eisenbud's *Commutative Algebra (with a View Toward Algebraic Geometry)* as the primary textbook. This book is incredibly long, and we will not come close to covering everything in this book. My current but ambitious goal is to get through sections 1-4, 6, and 8 of this book; this will likely be adapted over the course of the term.
- Another classic book is *Introduction to Commutative Algebra* by Sir Michael Atiyah and Ian MacDonal. This book, on some level, is the polar opposite of Eisenbud: it is very concise and relagates a large amount of the material to the exercises. It is a good book to have on hand especially if you plan on continuing to study commutative algebra and algebraic geometry, but by no means is this a required book for the course.

Additionally, here are some useful links:

- [This](#) is a link to the LEARN site for this course; I will upload all material there.
- [This](#) is a small site that I have put together that will contain a bunch of files and links. By no means is it a course website but it is a course repository that will be updated.

Prerequisites/Corequisites

This is a course that is a natural continuation of PMATH 347 (Groups and Rings), continuing on with the topics discussed there. Additionally, the topics in this course are at least related to the topics covered in PMATH 348 (Fields and Galois Theory). There is no way around having 347 (or an equivalent course at a different university) as a prerequisite to take this course, and due to both the algebraic and more general mathematical sophistication of a lot of the ideas here, 348 is a corequisite course.

Course Structure

The main learning component of this course will be lectures delivered through through a platform to be determined (the first thing I want to try is Google Meet but other options are available too). The lectures will involve me writing on my tablet and sharing this to everyone through said platform. The exact timing of the lectures is to be determined.

There will also be a couple office hours per week, which will have the same format. Again, the timing of the office hours is to be determined.

Homework and Tests

Grades in this course will be broken down as follows (numbers are liable to change between now and the start of the course):

- 70% of your grade comes from the assignments.
- 30% of your grade comes from the final project.

Assignments

There will be assignments given throughout the course. These will constitute the bulk of your grade and are extremely important for understanding the material. These will be a mixture of problems from Eisenbud and other problems assigned by me.

You are allowed to (and even encouraged to) discuss the assignments with other people in the class. However, when it comes to writing things up, this should be done by yourself. A large part of mathematics is communicating with others and gaining understanding from others, but it is important that you personally understand the material.

I strongly insist that people turn assignments in on time. Assignments will be submitted through crowdmark and assignments submitted late will receive zero marks. If you are having technical issues, please email me at least 15 minutes before the assignment is due on crowdmark.

Additionally, I am going to ask that you put no identifying information on your assignment submission. While I personally trust everyone involved, it is likely true that a nontrivial portion of the class knows the TA for this class and I want to avoid even the appearance of impropriety.

Final Project

There will not be a final exam in this class; between the difficulty of administering one during this pandemic and the topic itself, I feel that it is much simpler to just not have one. Instead, there will be a final project. Everyone will be expected to write up a 7-10 page paper³ on a more advanced topic. Additionally, students in 646 will be expected to give a short presentation in class on their topic⁴. As a corollary of this, I will ask that 646 students choose distinct topics for the final project.

Additionally, some of the topics might be larger in scale. If anyone is interested, I am happy to have two students work together on a project that has a slightly larger scope; I will expect a longer paper for such a collaboration (~15-20 pages).

A partial list of topics for the final project is here (this list will be updated at some point in the future):

- Completions
- The Artin-Rees Lemma (there could be two projects here: one giving the statement and proof, and one giving a geometric interpretation)
- Artinian Rings
- Dedekind Domains
- Introduction to Homological Algebra
- The Quillen-Suslin Theorem
- Path Algebras
- Group Rings
- Gröbner Bases

³For the love of god, please use LaTeX. If you are unfamiliar with LaTeX, I am more than happy to get you set up with some resources to get started.

⁴This is the only difference on my end between 446 and 646.

Tentative Schedule

Due to the nature of this term, I cannot commit to even a tentative schedule. I would like flexibility in being able to take more or less time on a topic as needed and a schedule will be more of a burden than a useful tool.

Having said that, there will be weekly announcements about what will be discussed that week so that you can read ahead and come prepared with questions.

Academic Integrity

In order to create and curate a culture of academic integrity, students are expected to promote honesty, fairness, respect, and responsibility. Students are expected to know the information [here](#).

Grievances

If a student feels that a decision affecting their university life has been unfair in any manner, then they may have grounds for a grievance. Policy 70 (and especially section 4), as seen [here](#), details the proper procedure for filing a grievance. If there is any doubt, contact the math department's administrative assistant, and they will provide further guidance.

Discipline

Students are expected to know what constitutes violations of academic integrity and accept responsibility for their actions. If a student is unsure about whether something constitutes a violation, wants help avoiding violations, or wishes to know course policies about group work and collaboration, they are encouraged to ask the course instructor, academic advisor, or undergraduate dean. For more information on offenses and types of penalties, Policy 71 (as seen [here](#)) provides a complete outline. For information on typical penalties, there are guidelines [here](#).

Appeals Process

Students may appeal decisions and penalties made under Policies 70 and 71 if there are grounds. If a student believes that they have a ground for an appeal should read Policy 72 (as seen [here](#)) for further guidance.

AccessAbility

The AccessAbility Office collaborates with all the academic departments at the University of Waterloo to arrange accommodations for students with disabilities without compromising the academic integrity of the course. If you require accommodations, you should register with the AccessAbility Office at the beginning of each term.

Currently, everything is physically closed, so the best ways to contact them are online at <https://uwaterloo.ca/accessability-services/>, via phone at (519) 888-4567 ext. 35082, and via email at access@uwaterloo.ca.

In the event that the university opens up, they can be found at Needles Hall, Room 1132.