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Brief article

Mathematics anxiety affects counting but not subitizing during visual enumeration

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ABSTRACT

Individuals with mathematics anxiety have been found to differ from their non-anxious peers on measures of higher-level mathematical processes, but not simple arithmetic. The current paper examines differences between mathematics anxious and non-mathematics anxious individuals in more basic numerical processing using a visual enumeration task. This task allows for the assessment of two systems of basic number processing: subitizing and counting. Mathematics anxious individuals, relative to non-mathematics anxious individuals, showed a deficit in the counting but not in the subitizing range. Furthermore, working memory was found to mediate this group difference. These findings demonstrate that the problems associated with mathematics anxiety exist at a level more basic than would be predicted from the extant literature.

1. Introduction

Basic numerical and mathematical skills have been shown to be crucial predictors of an individual's life success. Understanding numbers and mathematics is so critical that a deficit in basic mathematical abilities has been found to have a greater negative effect on employment opportunities than reading difficulties (Bynner & Parsons, 1997). Impairments in mathematical skills can result from a number of factors. One disorder associated with mathematical difficulties is mathematics anxiety, which is defined as a condition in which individuals experience negative affect when engaging in tasks demanding numerical and mathematical skills (Richardson & Woolfolk, 1980). Across a number of studies, individuals high in mathematics anxiety (HMA) have been shown to perform worse than their low-mathematics anxious (LMA) peers in solving difficult mathematical problems (Ashcraft & Kirk, 2001; Ashcraft, Kirk, & Hopko, 1998; Ashcraft, Krause, & Hopko, 2007).

The negative effects of MA and the potential mechanisms underlying these effects have been studied in some detail within the domain of mathematical problem solving. Ashcraft and colleagues (Ashcraft & Faust, 1994; Faust, Ashcraft, & Fleck, 1996) found that math anxiety had little effect on simple addition and multiplication problems. The solving of more complex arithmetic problems (e.g., arithmetic with carrying), however, was affected by MA. The most dominant theory of MA, posited by Ashcraft and colleagues, claims that MA individuals have difficulty with complex mathematical problem solving because MA induced ruminations occupy their working memory (WM; see Ashcraft, 2002). Thus, this theory of MA has two critical components. The first is that MA only affects complex mathematics; the second is that MA consumes WM resources that would otherwise be devoted to solving mathematical problems during calculation.

While there exist numerous demonstrations that MA individuals have difficulty with complex mathematical problem solving, there has been no exploration of whether

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the deficits extend to basic numerical processing skills, such as enumeration. Recent studies of individuals with mathematical disabilities (i.e., developmental dyscalculia) indicate that, in addition to having trouble with higher-level mathematical processing tasks, these individuals exhibit deficits in basic number processing tasks, such as magnitude comparison and enumeration (e.g., Landerl, Bevan, & Butterworth, 2004). This result is important because it suggests the possibility that the deficits seen in higher-level math may arise due to deficits in lower-level numerical processing skills. In the present study, we sought to determine whether or not mathematical processing difficulties observed in MA are accompanied by basic numerical processing deficits.

To test whether MA individuals have basic numerical processing deficits, a visual enumeration task was used. In this task participants are presented with a display containing multiple objects and are instructed to identify the number of objects presented. When enumerating visually presented objects, two distinct patterns of performance emerge. For 1–4 items, performance is fast and accurate with only a small increase in response times (RTs) and typically no decrease in accuracy as a function of the increase in the number of stimuli presented. This is commonly called ‘subitizing’ (Kaufman, Lord, Reese, & Volkmann, 1949). Conversely, for 5+ items, RTs increase and accuracy decreases as the number of stimuli presented increases (e.g., Trick & Pylyshyn, 1993). This is referred to as counting. A deficit in either the subitizing or counting range among individuals with MA would provide evidence that HMA individuals not only have a difficulty with high level mathematical processing but also a difficulty with basic numerical processing.

In addition, while the visual enumeration task is commonly used to index the presence/absence of a numerical processing deficit (e.g., with developmental dyscalculics; Landerl et al., 2004), it has the added benefit that the two numerical processing skills are thought to differentially tap working memory (WM). Specifically, counting is thought to put greater demands on WM than subitizing (Tuholski, Engle, & Baylis, 2001). This allows us to test the WM component of Ashcraft and colleagues’ theory.

In summary, Ashcraft and colleagues’ theory predicts that we should not observe any effects of MA on basic numerical processing, in either the counting or subitzing range. Ashcraft and colleagues also posit that the more WM demanding a math task is, the more susceptible performance on that task is to the effects of MA. Thus, a deficit in the counting range but not in the subitizing range would be consistent with the second part of Ashcraft and colleagues’ theory. On the other hand, an effect in the subitizing range would challenge both components of this theory.

2. Methods

2.1. Participants

Twenty-eight undergraduate students (16 female, 14 low and 14 high MA) from the University of Waterloo participated and were either granted experimental credit or were paid $6.00.

2.2. Stimuli, apparatus and procedure

The data were collected on a computer running E-Prime 1.1 (Schneider, Eschman, & Zuccolotto, 2001). Stimuli were displayed on a 17″ monitor. Responses were collected using a microphone headset. Each trial began with a fixation point presented for 500 ms. A display containing from one to nine square boxes was then centrally presented at fixation until a vocal response was detected. Participants were instructed to say aloud the number of squares on the screen. All squares were black on a white background. The individual area, total area, and density of the squares were varied to ensure that participants could not use non-numerical cues to make a correct decision (see Holloway & Ansari, 2009 for a complete description). There were a total of 378 trials.[1]

In addition, after performing the enumeration task, two measures of WM capacity were administered (a backwards digit span task, BDS, and a backwards letter span task, BLS; Wechsler, 1997). In these tasks participants heard a series of letters or digits presented at a rate of approximately one item per second. Participants then had to report the items back to the experimenter in the reverse order. The test continued with the addition of one item every second trial until participants made errors on two trials in a row. The participant’s score was the highest number of digits on which they made no errors.

Mathematics anxiety was measured using the Abbreviated Math Anxiety Scale (AMAS; Hopko, Mahadevan, Bare, & Hunt, 2003). Scores on the AMAS range from 9–45 with a higher score being indicative of a higher-level of MA. Participants were administered the AMAS during a mass testing session occurring approximately 2.5 months prior to our experiment. We selected participants with scores under 20 to constitute our low MA group and participants with scores over 30 to constitute our high MA group. These groups constituted roughly the top and bottom quartiles (24.4% and 26%) of the overall distribution.

3. Results

Trials on which there was a mistrial (2.1%) were removed prior to analysis. The data from one participant was discarded and replaced by another participant due to high error rates (63% incorrect).

3.1. Reaction times and errors

Fig. 1 depicts the relation between mean response times (ms) and number of items presented for the HMA and LMA groups. Trials on which there was an incorrect response (5.6%) were removed prior to RT analysis. The remaining RTs were submitted to a recursive data trimming proce-

[1] Items were presented six times each block with the exception of the numbers three and four which were presented one less and one more time respectively. Analyses controlling for the differences in the number of presentations for three and four did not change the pattern of results.

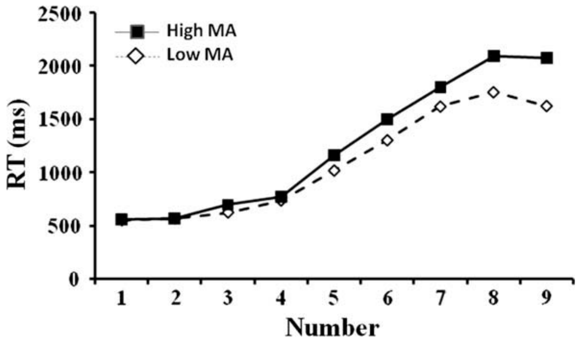

Fig. 1. Relation between mean response times (ms) and number of items presented for the high mathematics anxiety (HMA) and low mathematics anxiety (LMA) groups.

Table 1
Relation between percentage error rate and number of items presented for the high mathematics anxiety (HMA) and low mathematics anxiety (LMA) groups.

	Number								
	1	2	3	4	5	6	7	8	9
Low MA	0.5	0.2	1.2	0.9	2.4	4.9	10.1	15.7	11.6
High MA	0.4	0.3	0.9	1.1	3.1	8.4	10.5	17.2	15.0

dure using a 2.5 standard deviation cut-off in each cell resulting in an additional 4.8% of the RT data being removed. Table 1 depicts associated error data.

A (Number: 1–9) × 2 (MA Group: high vs. low) ANOVA yielded a main effect of number, $F(8, 208) = 182$, $MSE = 4891$, $p < 0.01$, $\eta^2 = 0.88$, and a marginal effect of MA Group, $F(1,26) = 3.9$, $MSE = 403685$, $p = 0.06$, $\eta^2 = 0.13$. Critically, there was a Number × MA Group interaction, $F(8, 208) = 3.4$, $MSE = 4891$, $p < 0.01$, $\eta^2 = 0.11$. A parallel ANOVA conducted on the error data yielded a main effect of number, $F(8, 208) = 14.9$, $MSE = 77$, $p < 0.01$, $\eta^2 = 0.37$, no effect of MA Group ($F < 1$) and no Number × MA Group interaction ($F < 1$). We next conducted separate ANOVAs on the subitizing and counting ranges.

3.1.1. Subitizing range

A 4 (Number: 1–4) × 2 (MA Group: high vs. low) ANOVA conducted on data within the subitizing range yielded a main effect of number, $F(3,78) = 58.0$, $MSE = 4169$, $p < 0.01$, $\eta^2 = 0.69$, but no effect of MA group ($F < 1$).[2] Critically, there was no Number × MA Group interaction ($F < 1$). A parallel ANOVA conducted on the error data yielded a main effect of number, $F(3,78) = 8.4$, $MSE = 0.01$, $p < 0.01$, $\eta^2 = 0.24$, but no main effect of MA Group ($F < 1$). There was no Number × MA Group interaction ($F < 1$).

3.1.2. Counting range

A 5 (Number: 5–9) × 2 (MA Group: high vs. low) ANOVA conducted on data within the counting range yielded a main effect of number, $F(4, 104) = 113$, $MSE = 31038$, $p < 0.01$, $\eta^2 = 0.81$, and no effect of MA Group, $F(1,26) = 2.9$, $MSE = 628879$, $p > 0.05$, $\eta^2 = 0.10$. Critically, there was a Number × MA Group interaction, $F(4, 104) = 2.8$, $MSE = 31038$, $p < 0.05$, $\eta^2 = 0.10$, in which the HMA group responded more slowly as a function of increasing number than the LMA group. A parallel ANOVA conducted on the error data yielded a main effect of number, $F(4, 104) = 10.7$, $MSE = 0.01$, $p < 0.01$, $\eta^2 = 0.29$, such that as number increased, so did the number of errors. There was no main effect of MA Group and no Number × MA Group interaction, (Fs < 1). Thus, the results do not represent a speed/accuracy tradeoff. There was no significant correlation between participants' overall RT and their percent errors.

3.2. Working memory and enumeration

There was no significant difference between the mean BDS of LMA participants (6.8) and HMA participants (6.3), $t(26) = 1.1$, $p > 0.05$, but there was a significant difference between the mean BLS of LMA participants (6.1 items) and HMA participants (5.0 items), $t(26) = 2.4$, $p < 0.05$. To determine the relation between WM and performance in the visual enumeration task we created a composite WM measure (WMC) comprising the average of each individual's BDS and BLS scores. We then conducted an analysis parallel to the initial analyses with WMC as a covariate.

3.2.1. Subitizing range

A 4 (Number: 1–4) × 2 (MA Group: high vs. low) ANCOVA with WMC as a covariate conducted on response time data within the subitizing range yielded a main effect of number, $F(3,75) = 3.5$, $MSE = 4228$, $p < 0.05$, $\eta^2 = 0.12$, but no effect of MA group ($F < 1$) and no Number × MA Group interaction ($F < 1$).

3.2.2. Counting range

A 5 (Number: 5–9) × 2 (MA Group: high vs. low) ANCOVA with WMC as a covariate yielded a main effect of number, $F(4, 100) = 4.9$, $MSE = 31373$, $p < 0.01$, $\eta^2 = 0.16$, and no effect of MA Group, $F(1,25) < 1$. Critically, the aforementioned Number × MA Group interaction is no longer significant, $F(4, 100) = 1.3$, $MSE = 31373$, $p > 0.05$, $\eta^2 = 0.04$, suggesting that WM differences between groups may mediate the different performance for HMA and LMA individuals.[3]

4. General discussion

The primary purpose of the present experiment was to determine whether individuals with MA have a basic numerical processing deficit in addition to their well-established mathematical processing impairments. Here we clearly show that in the context of a visual enumeration task, HMA individuals perform significantly worse in the counting range than LMA individuals. This numerical deficit was not found to extend to the subitizing range.

[2] We have replicated the RT and accuracy results reported here in a second experiment conducted in our lab.

[3] If the subitizing range is defined as 1–3 the results are qualitatively similar.

The present results have important implications for current theoretical understanding of mathematics anxiety. The dominant account of mathematics anxiety has at its core two main ideas. Ashcraft and colleagues claim that (1) MA individuals only have difficulty with complex mathematics (such as multi-digit arithmetic problems) and (2) MA impairs performance by temporarily depleting WM resources. The data presented here significantly challenges the first claim. Contrary to Ashcraft and colleagues claim, individuals with MA do, in fact, have a basic numerical processing deficit. Thus the present findings directly challenge existing accounts of MA by revealing that, contrary to existing hypotheses, the effects of MA extend to numerical processing tasks that are more basic than single-digit arithmetic. It should be noted, however, that while Ashcraft and colleagues failed to find a difference in single-digit arithmetic, it is possible that their null result was due to a lack of statistical power and/or subtle strategy effects in single-digit arithmetic. For example, contrasting multiplication and subtraction, which are known to draw on different cognitive processes and strategies (Dehaene, Piazza, Pinel, & Cohen, 2003), may unmask underlying numerical processing deficits.

This discovery, that MA affects processing at a much more basic level than previously thought, leads to a potential reconceptualization of the developmental trajectory of MA. Specifically, the present results suggest that MA could result from a basic level deficit in numerical processing that compromises the development of higher-level mathematical problem solving. This basic level deficit could result in difficulties in math tasks resulting in negative experiences with math and as a result anxiety when having to engage in math related tasks. The hypothesis that MA results from a basic deficit in numerical processing, while not new, had been largely abandoned in recent years (see Ashcraft et al., 2007). Thus, research aimed at developing a better understanding of this putative early deficit, for example, by investigating MA effects on tasks held to index fundamental mathematical or number processes are needed.

The second core idea concerning MA is that it leads to a decrease in WM capacity when MA individuals are performing math tasks. According to this account, performance deficits observed in MA are caused by anxiety-induced ruminations that limit the WM capacity available to perform mathematical tasks. As such, this account predicts an effect of MA in the WM demanding counting range but not in the non-WM demanding subitizing range. The dissociation reported here between subitizing and counting is thus consistent with the second component of Ashcraft and colleagues' theory.

Additional support for Ashcraft and colleagues was provided by our observation that when differences in WM capacity were controlled for, the difference in performance in the counting range was eliminated. However, this apparent support needs to be taken with a grain of salt. Ashcraft and colleagues posit a *transitory* effect of MA on WM capacity such that while performing a math related task, MA consumes WM resources that would otherwise be devoted to solving the numerical and mathematical problems (see also Eysenck & Calvo, 1992). Thus, when HMA and LMA individuals are not performing math related tasks, their WM capacity should be equivalent and WM deficits should only appear on math tasks. However, here we found a difference on a non-math related WM task. Interestingly, Ashcraft and Kirk (2001) also show a small non-math related difference in WM capacity on a listening span task (Salthouse & Babcock, 1990). MA would, as Ashcraft and colleagues claim, then further exacerbate their WM deficit.

4.1. Accounts of systems of enumeration

In addition to demonstrating that individuals with MA have deficits in numerical processing and that these deficits are related to WM, the present results also have implications for our understanding of number processing in general. Specifically, there exists a long-standing debate regarding whether or not we employ the same underlying processes while subitizing and while counting. According to one view subitizing and counting rely on the same representational mechanisms (Balakrishnan & Ashby, 1991; Dehaene & Changeux, 1993; Gallistel & Gelman, 1992). An alternative account postulates the existence of different cognitive mechanisms, one dedicated to small sets of objects and one deployed during the enumeration of large sets of objects (Feigenson, Dehaene, & Spelke, 2004; Revkin, Piazza, Cohen, & Dehaene, 2008). While the present study was not designed to discriminate between these two accounts, the fact that MA influenced performance in the counting range but not in the subitizing range is consistent with the latter account.

5. Conclusion

We have demonstrated, using a visual enumeration task that HMA individuals differ from their LMA peers on the enumeration of items in the counting but not the subitizing range. Furthermore, the present findings reveal that these differences appear to stem from differences in WM capacity. These data are taken as evidence that the effect of MA extends beyond the level of mathematical processing and into that of basic numerical processing and that even these relatively low-level deficits are likely mediated by WM.

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