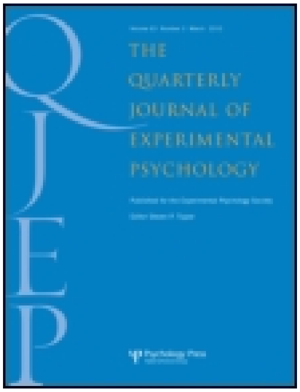

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The effect of mathematics anxiety on the processing of numerical magnitude

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In an effort to understand the origins of mathematics anxiety, we investigated the processing of symbolic magnitude by high mathematics-anxious (HMA) and low mathematics-anxious (LMA) individuals by examining their performance on two variants of the symbolic numerical comparison task. In two experiments, a numerical distance by mathematics anxiety (MA) interaction was obtained, demonstrating that the effect of numerical distance on response times was larger for HMA than for LMA individuals. These data support the claim that HMA individuals have less precise representations of numerical magnitude than their LMA peers, suggesting that MA is associated with low-level numerical deficits that compromise the development of higher level mathematical skills.

Keywords: Numerical distance effect; Numerical comparison; Numerical cognition; Mathematics anxiety.

Mathematics anxiety (MA) is a condition in which individuals experience negative affect when engaging in tasks demanding numerical and mathematical skills (Richardson & Woolfolk, 1980). Across a number of studies, individuals high in mathematics anxiety (HMA) have been shown to perform worse than their low mathematics-anxious (LMA) peers on a wide range of numerical and mathematical tasks, ranging from simple tasks such as counting objects (Maloney, Risko, Ansari, & Fugelsang, 2010) to more complex arithmetic problems involving carrying (Ashcraft & Faust, 1994).

The most dominant theory explaining MA, posited by Ashcraft and colleagues (e.g., Ashcraft & Kirk, 2001), asserts that HMA individuals suffer from MA-induced ruminations, which compromise the availability of working memory (WM) resources, which in turn leads to lower performance on numerical and mathematical tasks. While previous research assumed that MA only affects highly WM-demanding maths, recent work by Maloney, Risko, Ansari, et al. (2010) suggests otherwise. Maloney et al. presented LMA and HMA participants with displays containing from 1–9 squares and asked them to

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enumerate the squares. They found that both groups enumerated equally quickly when there were anywhere from 1–4 squares presented. However, HMA individuals were significantly slower to enumerate from 5 to 9 squares than their LMA peers. Based on these results, Maloney et al. posited that MA could result from a basic low-level deficit in numerical processing that compromises the development of higher level mathematical problem solving. According to this conceptualization, WM deficits may play a secondary role. That is, the effects of MA that have been observed to date may be constrained by low-level deficits in numerical magnitude processing and that the anxiety-induced WM reduction serves to further exacerbate the effects of low-level deficits. However, Maloney, Risko, Ansari, et al. (2010) were agnostic with regards to the precise nature of such a low-level deficit. In the present investigation we explore the prediction that MA is associated with deficits in the representation of numerical magnitude.

In the numerical cognition literature it is frequently posited that numerical magnitudes are represented mentally on an internal number line and that this mental number line underlies "number sense"—that is, the fundamental ability to automatically and efficiently process numerical magnitude information (Dehaene, 1997). Each number is thought to hold a specific place on the number line and share representational features with the numbers close to it. Furthermore, the degree of overlap between numerical magnitude representations is thought to vary between individuals. For example, someone with a precise representation of number will have relatively less overlap between close numbers than will an individual who exhibits less precise numerical representations (Dehaene, Dehaene-Lambertz, & Cohen, 1998; Holloway & Ansari, 2009).

The processing of numerical magnitude is most commonly assessed using different variants of the numerical comparison task. For example, participants may be asked to compare a single digit to a fixed standard, or to compare two digits presented simultaneously. While there are different variants of the task, all yield a similar pattern of data. This pattern is known as the numerical distance effect (NDE), referring to an inverse relationship between numerical distance and response times. In other words, participants are faster and more accurate at indicating which of two numbers is larger (or smaller) when the numerical distance separating the two numbers is relatively large (e.g., 2 vs. 9) than when it is comparatively small (e.g., 8 vs. 6; Dehaene, Dupoux, & Mehler, 1990; Maloney, Risko, Preston, Ansari, & Fugelsang, 2010; Moyer & Landauer, 1967).

The most dominant theoretical model of the NDE posits that this effect reflects the relative overlap of numerical magnitude representation on a mental number line, where numbers that are positioned closely to one another share more representational overlap and are thus harder to discriminate during number comparison than those that are far apart.[1] Importantly, individual differences in the NDE have been associated with variability in mathematical skills. For example, Holloway and Ansari (2009) demonstrated, in a group of 6–8-year-old children, a relation between an individual's NDE in a symbolic numerical comparison task and their performance on a standardized test of mathematical skills. Specifically, participants with larger NDEs (and presumably a less precise representation of number) displayed poorer performance on a maths task than did those with smaller NDEs (and presumably a more precise representation of number).

[1] An alternative account is that the NDE indexes the *comparison process* involved in numerical comparison, rather than *numerical representation* per se (e.g., Van Opstal, Gevers, De Moor, & Verguts, 2008). While we acknowledge that there is presently a debate with respect to what the NDE indexes, it is not important for the present investigation whether the NDE is an index of numerical representation or numerical comparison processes. Rather, what is important is that we accept that the NDE is indexing numerical magnitude at a very basic level. Nonetheless, the present paper will adhere to the widely accepted view that the NDE indexes overlap of numerical representations in order to facilitate the communication of the findings.

The present investigation

In view of the above data suggesting that the NDE can serve as a measure of individual differences in the processing and representation of numerical magnitude, the present investigation examined both HMA and LMA individuals' performance on two symbolic variants of the numerical comparison task. If HMA individuals do, in fact, have less precise representations of numerical magnitude than their LMA peers, then HMA individuals should exhibit a larger NDE than those with LMA.

EXPERIMENT 1

Method

Participants
Forty-eight undergraduate students (24 LMA and 24 high HMA) from the University of Waterloo participated and either were granted experimental credit or were paid $5.00. MA was measured using the Abbreviated Math Anxiety Scale (AMAS; Hopko, Mahadevan, Bare, & Hunt, 2003). Scores on the AMAS range from 9–45. Participants were administered the AMAS during a mass testing session occurring approximately 2 months prior to our experiment. We selected participants with scores under 20 to constitute our LMA group and participants with scores over 30 to constitute our HMA group. These groups constituted roughly the top and bottom quartiles (24.4% and 26%) of the overall distribution.

Stimuli, apparatus, and procedure
The data were collected on a Pentium 4 PC computer running E-Prime 1.1 (Schneider, Eschman, & Zuccolotto, 2001). Stimuli were displayed on a 19″ monitor. Numerical distance was measured using a lower/higher than five (L/H5) variant of the numerical comparison task. Each trial began with a fixation point that remained on the screen for 500 ms. A display containing a single Arabic digit in 18-point Arial font was presented at fixation. Numbers ranged from 1–4 and from 6–9. Participants were told to identify whether the presented number was lower than five or higher than five by pressing the "A" key to denote lower and the "L" key to denote higher. The number remained on the screen until the participants made a button press. There were a total of 160 trials. The numerical distance between the stimuli and the number 5 ranged from 1 to 4, with 40 comparison trials total per distance. Stimulus displays were presented randomly.

Results and discussion

Response times (RTs) and errors were analysed across participants with numerical distance as a within-subject variable and MA as a between-subject variable. Trials on which there was an incorrect response were removed prior to RT analysis (2.7%). The remaining RTs were submitted to a 2.5-*SD* (where *SD* is standard deviation) recursive outlier removal procedure (Van Selst & Jolicoeur, 1994), resulting in the removal of an additional 4.0% of the data.

A 4 (distance: 1–4) × 2 (MA group: low vs. high) analysis of variance (ANOVA) conducted on the RT data yielded a main effect of distance, $F(3, 138) = 28.9$, $MSE = 1{,}087.3$, $p < .01$, $\eta^2 = .39$, and a main effect of MA group, $F(1, 46) = 5.0$, $MSE = 312{,}233.4$, $p < .05$, $\eta^2 = .10$, whereby the HMA group were slower overall to respond than the LMA group. Importantly, there was a Distance × MA Group interaction, $F(3, 138) = 2.8$, $MSE = 1{,}087.3$, $p < .05$, $\eta^2 = .06$, whereby the effect of distance was larger for the HMA group than for the LMA group. A *t* test on the slopes of the distance effects for HMA and LMA participants supported this interpretation by revealing a steeper NDE for the HMA group than for the LMA group, $t(46) = 2.0$, $p = .05$. A parallel analysis on the error data yielded a main effect of distance, $F(3, 138) = 15.0$, $MSE = 126.8$, $p < .01$, $\eta^2 = .32$, no main effect of MA group, $F < 1$, and no interaction, $F(3, 138) = 1.1$, $MSE = 126.8$, $p > .05$, $\eta^2 = .01$. (See Figure 1.)

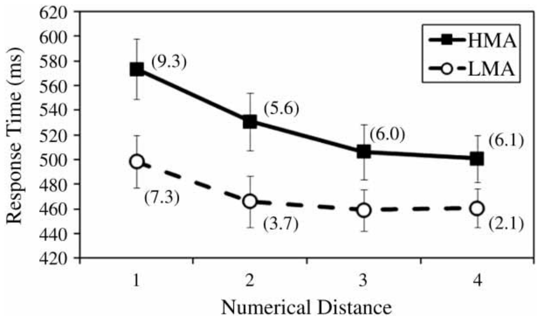

Figure 1. *Mathematics anxiety by numerical distance in the L/H5 (lower/higher than five) symbolic comparison variant (errors in parentheses). HMA = high mathematics anxiety. LMA = low mathematics anxiety. The error bars depict the standard error of the mean.*

The primary purpose of the present experiment was to determine whether individuals with HMA represent numbers differently from their LMA peers. Here we show that in the context of a L/H5 task, numerical distance interacts with MA whereby the effect of numerical distance was larger for HMA than for LMA individuals. There are several variations of the numerical comparison task—for example, participants may also be asked to compare one digit to a set standard or to state whether two simultaneously presented digits are the same or different. While these variants all produce the characteristic NDE, we, Maloney, Risko, Preston, et al. (2010), have recently found that some variants of the numerical comparison task are more reliable than others. Furthermore, we identified that the NDE found using the L/H5 variant does not strongly correlate with the NDE found using a simultaneous (two digits presented side by side) comparison variant, suggesting that these variants might be differentially indexing numerical magnitude. Against this background, we wanted to ascertain that the MA by NDE interaction generalizes to a different variant of the numerical comparison task. Additionally, it is possible that the L/H5 variant of the number comparison task may require more WM resources than other variants, given that in the L/H5 variant one must keep the standard (5) online in WM while performing the task. As such, using the L/H5 variant makes it difficult to rule out the possibility that WM is playing a role in the observed MA effects. To address these concerns, we conducted a follow-up experiment that employed a simultaneous two-digit variant of the numerical comparison task.

EXPERIMENT 2

Method

Participants

Forty-four undergraduate students (22 low and 22 high MA) from the University of Waterloo participated and either were granted experimental credit or were paid \$6.00.

Stimuli, apparatus, and procedure

The stimuli, apparatus, and procedure (i.e., trial composition) were identical to those used in Experiment 1, with the following exception. Following the 500-ms fixation point, a display containing two Arabic digits was presented. Participants were told to identify which of the two numbers was numerically larger by pressing the "A" key to denote that the number on the left was larger and the "L" key to that the number on the right was larger.

Results and discussion

RTs and errors were analysed across participants with numerical distance as a within-subject factor. Trials on which there was an incorrect response were removed prior to RT data analysis (3.2%). The remaining RTs were submitted to a 2.5-*SD* recursive outlier removal procedure (Van Selst & Jolicoeur, 1994), resulting in the removal of an additional 3.1% of the data.

A 4 (distance: 1−4) × 2 (MA group: low vs. high) ANOVA conducted on the RT data yielded a main effect of distance, $F(3, 126) = 53.3$, $MSE = 574.1$, $p < .01$, $\eta^2 = .56$, and no main effect of MA group, $F < 1$. Importantly, replicating the key finding in Experiment 1, there was a Distance × MA Group interaction, $F(3, 126) = 2.7$, $MSE = 574.1$, $p < .05$, $\eta^2 =$

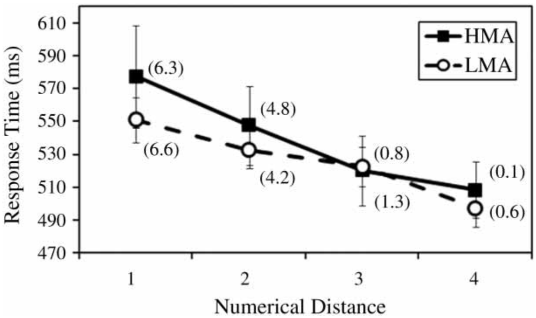

Figure 2. *Mathematics anxiety by numerical distance in the two digit symbolic comparison variant (errors in parentheses). HMA = high mathematics anxiety. LMA = low mathematics anxiety. The error bars depict the standard error of the mean.*

.06, whereby the NDE was larger for HMA than for LMA participants. A *t* test on the slopes of the distance effects for HMA and LMA revealed a marginally steeper NDE for the HMA group than for the LMA group, $t(42) = 1.8$, $p = .07$. A parallel analysis on the error data yielded a main effect of distance, $F(3, 126) = 32.3$, $MSE = 111.3$, $p < .01$, $\eta^2 = .44$, no main effect of MA group, and no interaction, Fs < 1. (See Figure 2.)

Experiment 2 clearly replicated the MA by NDE interaction from Experiment 1. Therefore, in a second experiment, using a different variant of the symbolic comparison task, with a new set of HMA and LMA participants, we demonstrate that size of the NDE is larger for HMA than for LMA participants.

EXPERIMENTS 1 AND 2 COMBINED ANALYSES

As noted as part of the motivation for Experiment 2 above, the L/H5 variant may be more WM demanding than the two-digit comparison variant. If this is the case, and if a decrease in WM capacity is what is driving the MA by distance interaction seen here, then we should see a larger effect of MA on numerical distance in the L/H5 variant than in the two-digit comparison variant. To test this possibility, we conducted a combined analysis of Experiments 1 and 2, including task variant as a between-subjects factor. This 2 (MA group: high vs. low) × 2 (task variant: L/H5 vs. two-digit comparison) × 4 (distance: 1–4) ANOVA yielded a main effect of distance, $F(3, 264) = 71.2$, $MSE = 842.4$, $p < .01$, $\eta^2 = .45$, a marginal effect of MA group, $F(1, 88) = 3.3$, $MSE = 33{,}396.2$, $p = .07$, $\eta^2 = .04$, no MA Group × Task Variant interaction, $F(1, 88) = 1.4$, $MSE = 33{,}396.2$, $p > .05$, $\eta^2 = .02$, and a Distance × MA Group interaction, $F(3, 264) = 4.7$, $MSE = 842.4$, $p < .01$, $\eta^2 = .05$. A follow-up *t* test on the slopes revealed that the slope of the NDE was larger for HMA than for LMA participants, $t(90) = 2.6$, $p = .01$. Importantly, there was no Distance × MA Group × Task Variant interaction, $F < 1$. Therefore, the results of the combined analysis indicate that the MA Group × Distance interaction does not vary as a function of task variant and, as such, does not vary as a function of the WM demands of the task.

GENERAL DISCUSSION

Currently, the dominant account of MA posits that MA results in anxiety-induced thoughts during mathematical problem solving and that these thoughts consume WM resources. The current literature thus assumes that the performance deficits seen in MA individuals are due to a decrease in WM capacity rather than a lack of mathematical skill per se. Therefore, the present findings of an interaction between MA and NDE on two variants of the symbolic numerical comparison task could be construed as problematic for the dominant theory of MA, as there is no published evidence as of yet to suggest that numerical comparison is particularly WM demanding. The present data are consistent, however, with the Maloney, Risko, Ansari, et al. (2010) finding that MA may result from a basic low-level deficit in numerical processing that compromises the development of higher level mathematical skills.

While these data do not allow us to explicitly state why or how MA develops and evolves, they do allow us to infer that less precise representations of numerical magnitude most likely play a role. It seems likely that a hybrid theory can best explain the data to date. Here, the difficulty that individuals with MA have with high-level mathematics may ultimately stem from less precise representations of numerical magnitude. Certainly, as a function of their difficulty with mathematics, people could develop MA, which in turn could result in WM-demanding ruminations, thereby exacerbating the difficulties these individuals already experience.

In order for the proposed account to be valid, one must accept that there is a link between the precision of magnitude representations (as indexed by the NDE) and high-level mathematical skills. Support for this link has come from a variety of sources. As noted in the introduction, Holloway and Ansari (2009) demonstrated that there exists a relation between one's symbolic NDE and fluency in mathematics, such that larger NDEs are associated with poorer fluency in mathematics (see also Mundy & Gilmore, 2009). Even more compelling are longitudinal data collected by De Smedt, Verschaffel, and Ghesquière (2009) showing that accuracy and speed of number comparison predict future mathematics achievement.

Further evidence for the link between the precision of magnitude representations and high-level mathematical skills comes from observations from other special populations. For example, children with developmental dyscalculia have been shown to perform more poorly on tasks of basic numerical magnitude processing than do typically developing children (Landerl, Bevan, & Butterworth, 2004; Mussolin, Mejias, & Noel, in press). Atypical performance on number comparison tasks has also been demonstrated in individuals with Williams syndrome (Paterson, Girelli, Butterworth, & Karmiloff-Smith, 2006) and chromosome 22q.11.2 deletion (Simon, Bearden, Mc-Ginn, & Zackai, 2005). Taken together, these findings indicate that basic processing of numerical magnitude is affected in children who present with mathematical difficulties; therefore, the origins of such deficits may lie in these foundational competencies. While the presence of a large NDE combined with mathematical difficulties in individuals with MA does not equate MA individuals with individuals with developmental dyscalculia, the results from the present study suggest that we can now add MA individuals to the growing list of special populations who present with both atypical performance on number comparison tasks and poor performance on complex mathematical tasks.

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