Unquantized anomalies in topological semimetals

Anton Burkov

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Outline

- Unquantized anomalies in topological semimetals.

- 3D Fractional Quantum Hall effect in magnetic Weyl semimetals.

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What is topological about topological semimetals?

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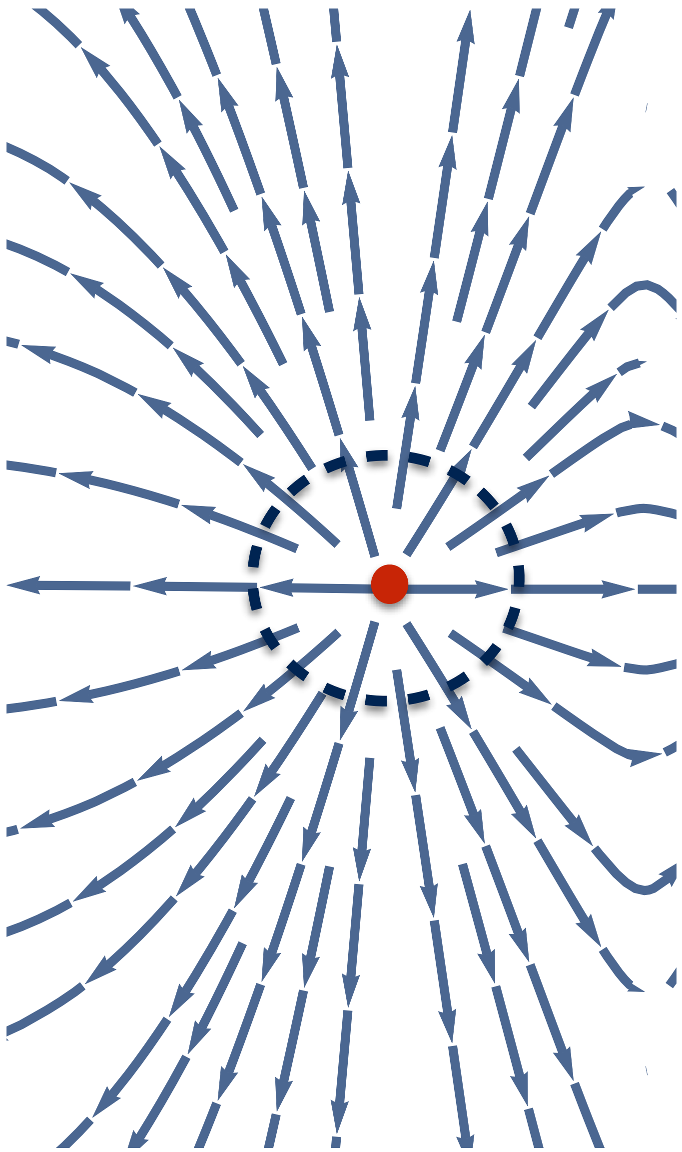
- Typically talk about topological invariants in momentum space.

$$\frac{1}{2\pi}\int \mathbf{\Omega}(\mathbf{k})\cdot d\mathbf{S} = C$$

$$\mathbf{\Omega}(\mathbf{k}) = \pm\frac{\mathbf{k}}{2k^3}$$



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- A more general picture is in terms of response: for example anomalous Hall effect in magnetic Weyl semimetals.

- But other kinds of topological semimetals don’t have any obvious “topological” response.

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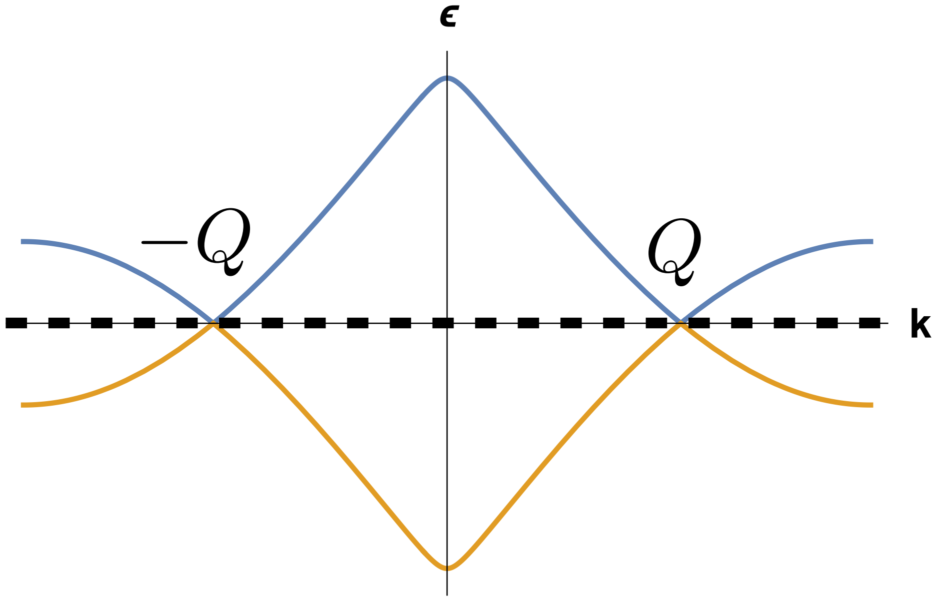
- Does it mean that they are not really "topological" in any sense once interactions are included?

- No, but the responses are more subtle.

Magnetic Weyl semimetal

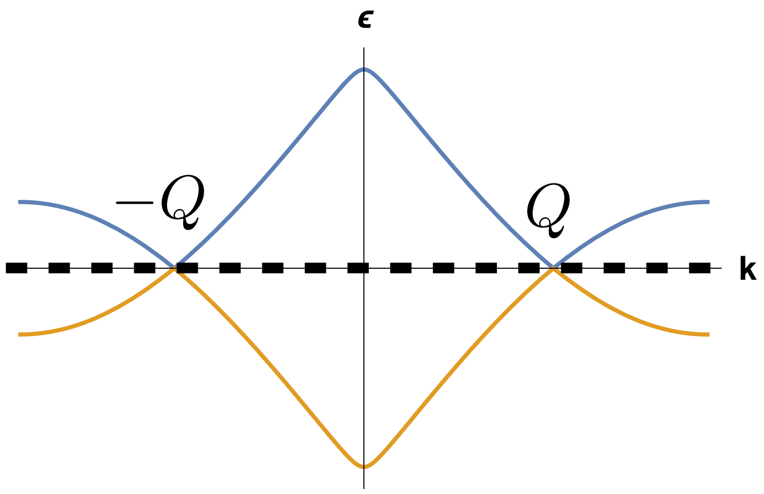


$$\hbar = c = e = 1$$

$$\sigma_{xy} = \frac{1}{2\pi}\frac{2Q}{2\pi}$$

Wiedemann-Franz law: $$\kappa_{xy} = \sigma_{xy}\left(\frac{\pi^2 k_B^2 T}{3}\right)$$

Magnetic Weyl semimetal



$$\sigma_{xy} = \frac{1}{2\pi}\frac{2Q}{2\pi}$$

$$S = -\frac{i\sigma_{xy}}{2}\int d\tau d^3 r \epsilon_{z\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

chiral anomaly

- Not invariant under large gauge transformations:

$$A_0 \to A_0 + \partial_\tau \chi \qquad \int_0^\beta \partial_\tau \chi = 2\pi \qquad S \to S + i\,2QL_z n$$

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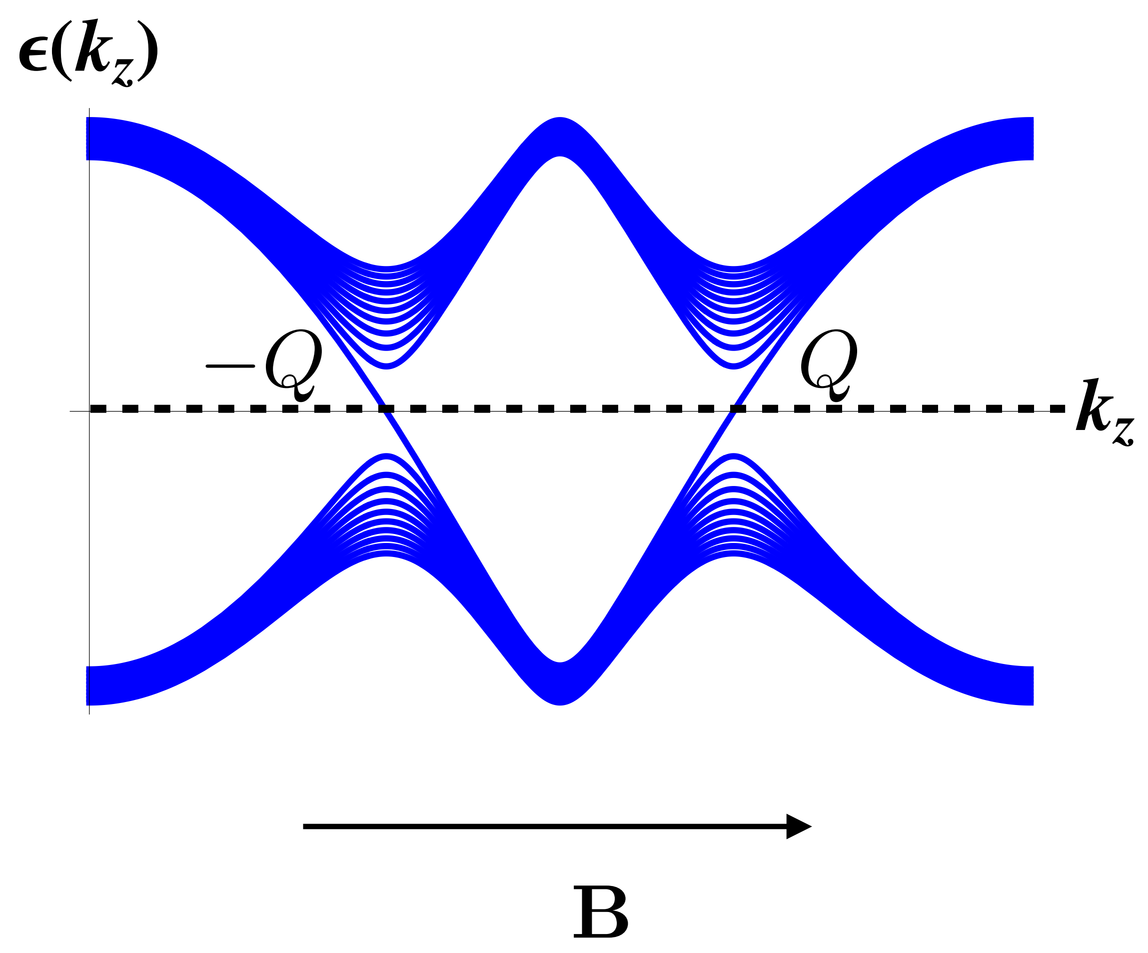
- In the absence of Fermi surface (excluded by filling) this makes Weyl nodes necessary.

Magnetic Weyl semimetal

$$S = -\frac{i\sigma_{xy}}{2}\int d\tau d^3r \epsilon_{z\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

- 2π flux line carries fractional charge per unit cell:

$$Q_{U(1)} = 2\pi\sigma_{xy}L_z = \frac{2Q}{2\pi}L_z$$



Magnetic Weyl semimetal

- 2π flux line carries fractional charge per unit cell:

$$\nu = \frac{2Q}{2\pi/d} = \frac{Q_{U(1)}}{N_z}$$

- This is a more "topological" property than Hall conductivity since Hall conductivity contains a length scale:

$$\sigma_{xy} = \frac{1}{2\pi}\frac{2Q}{2\pi} = \frac{\nu}{2\pi d}$$

- Unlike Hall conductivity, this is also generalizable to other topological semimetals.

Topological term

$$S = -i\frac{1}{2}\frac{\nu}{2\pi d}\int d\tau d^3 r \epsilon_{z\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

- Consider deformed crystal: $\frac{1}{d} \to \frac{1}{d}\left(\delta_{\mu z} - \partial_\mu u_z\right)$

Cortijo et al. Pikulin et al. Grushin et al.

Topological term

$$S = -i\frac{1}{2}\frac{\nu}{2\pi d}\int d\tau d^3 r \epsilon_{z\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

- Consider deformed crystal: $\frac{1}{d} \to \frac{1}{d}\left(\delta_{\mu z} - \partial_\mu u_z\right)$

- Account for the fact that atomic position is only defined modulo primitive translation:

$$\partial_\mu u_z \to \partial_\mu u_z + z_\mu \qquad \int_C \partial_\mu u_z = n$$

Nissinen & Volovik   Song et al.   Manjunath & Barkeshli

Topological term

- Focus on topological part of the action, which involves only gauge fields:

$$S = -\frac{i}{2}\frac{\nu}{2\pi}\int z \wedge A \wedge dA$$

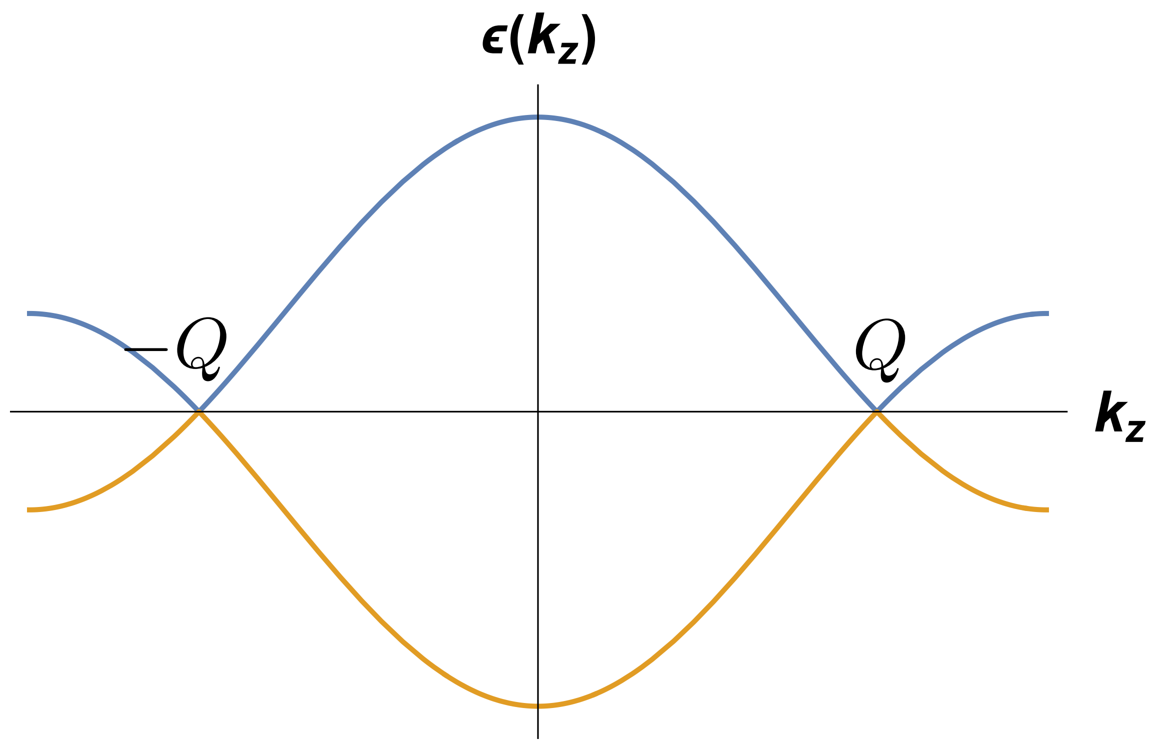
- ν is charge per unit cell on the 2π flux line.

$\int_z z = N_z$ number of unit cells along z.

$\int_{x,y,t} z$ shear of the periodic boundary conditions in x,y,t along z.

$\int_C z = n$ number of dislocation lines enclosed by C.

Generalization to Dirac semimetal



Cd_3As_2

Na_3Bi

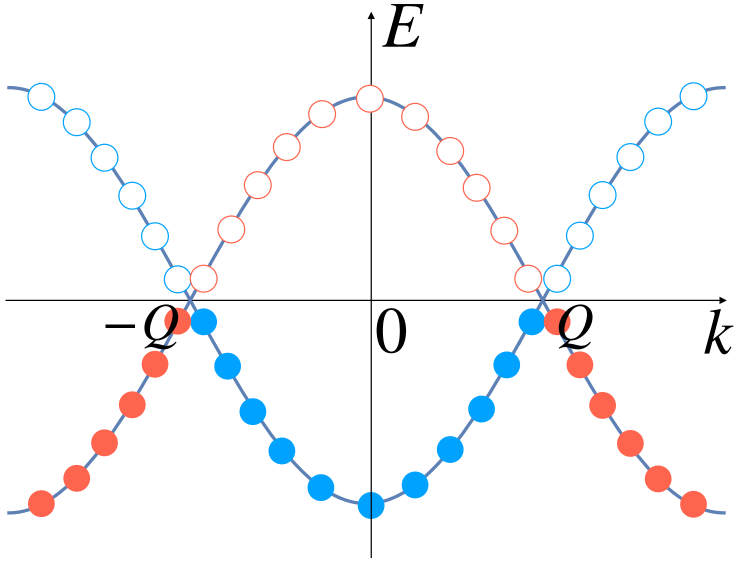
- Pair of Dirac nodes on a rotation axis, protected by C_4 symmetry.

- Topological term should involve U(1), translation, and rotation gauge fields.

Lowest Landau level

- Can infer topological term by examining rotation operator eigenvalues in the LLL.

$$R_4 = e^{\frac{i\pi}{4}} e^{-\frac{i\pi}{4}(2-\sigma^z)s^z}$$



$$C_{4\uparrow} = e^{i0} \qquad C_{4\downarrow} = e^{i\pi}$$

$$Q_{C_4} = \pi \nu N_z \frac{BL_xL_y}{2\pi} \qquad \nu = \frac{2Q}{2\pi/d}$$

Topological term

- Can infer topological term by examining rotation operator eigenvalues in the LLL.

$$Q_{C_4} = \pi \nu N_z \frac{B L_x L_y}{2\pi}$$

$$S = -i\frac{\nu}{2}\int z \wedge c_4 \wedge dA$$

- Rotation gauge field accounts for local rotations of the coordinates:

Topological term

- Can infer topological term by examining rotation operator eigenvalues in the LLL.

$$Q_{C_4} = \pi \nu N_z \frac{B L_x L_y}{2\pi}$$

$$S = -i\frac{\nu}{2}\int z \wedge c_4 \wedge dA$$

- Temporal component of c_4 couples to the rotation charge.

- For any xy-cycle enclosing a π/2 disclination line in the z-direction:

$$\int_{C_{xy}} c_4 = 1$$

Topological term

$$S = -i\frac{\nu}{2}\int z \wedge c_4 \wedge dA$$

- At low energies Dirac semimetal may be viewed as a time-reversed pair of Weyl semimetals:

$$S = -i\frac{\nu}{4\pi}\int z \wedge (A + \pi c_4) \wedge d(A + \pi c_4) + i\frac{\nu}{4\pi}\int z \wedge (A + 0 c_4) \wedge d(A + 0 c_4)$$

Topological term

$$S = -i\frac{\nu}{2}\int z \wedge c_4 \wedge dA$$

- Not invariant under gauge transformation:

$$c_4 \to c_4 + 4\alpha \qquad \alpha \in \mathbb{Z}$$

$$S \to S + 4\alpha\pi i\nu N_z n$$

- This makes gapless Dirac points necessary, except for certain values of ν.

Charge on disclination

$$S = -i\frac{\nu}{2}\int z \wedge c_4 \wedge dA$$

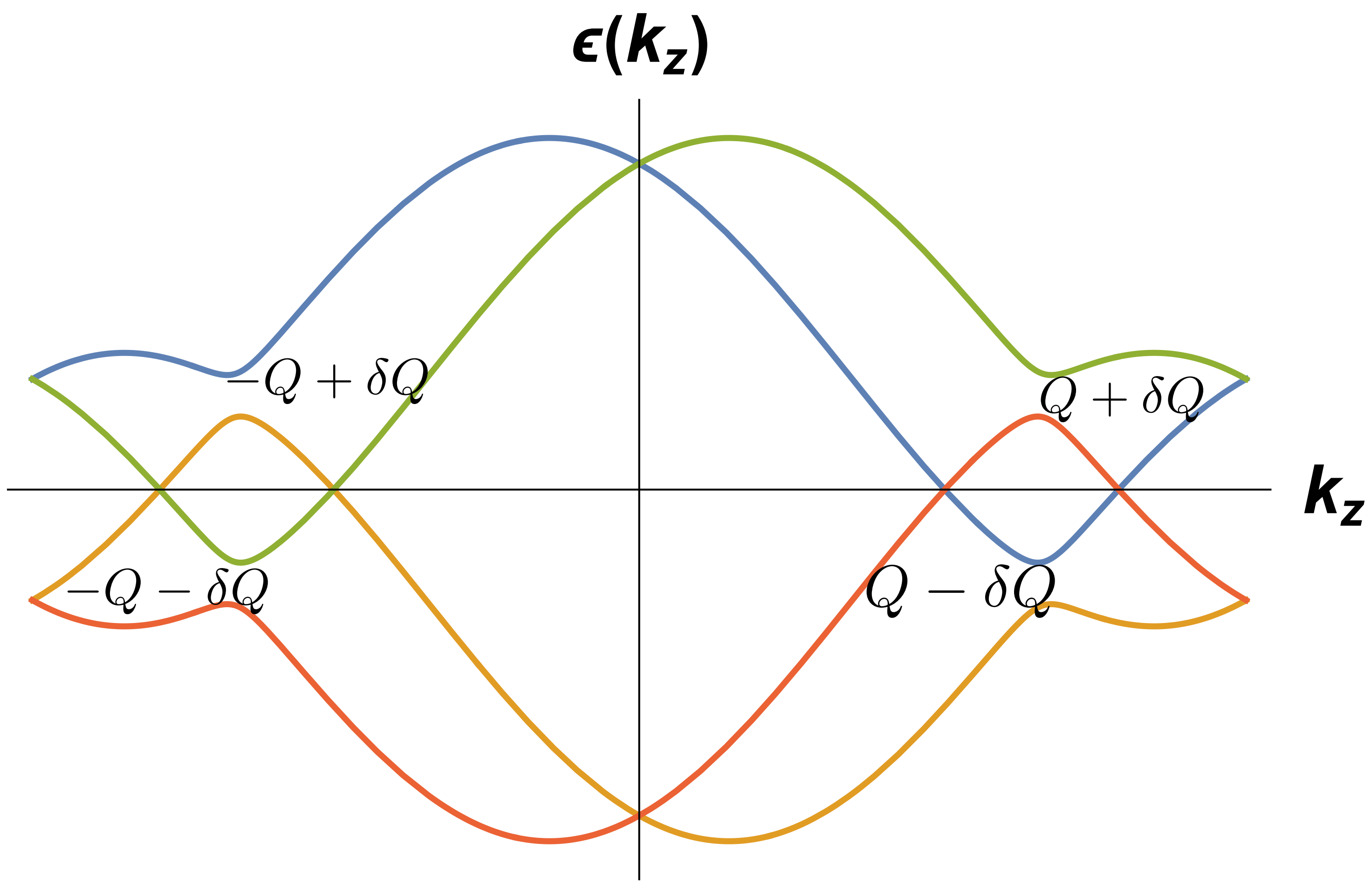
- A nontrivial consequence is that disclination along z direction carries a noninteger charge per unit cell:

$$Q_{U(1)} = \frac{\nu}{2} N_z$$

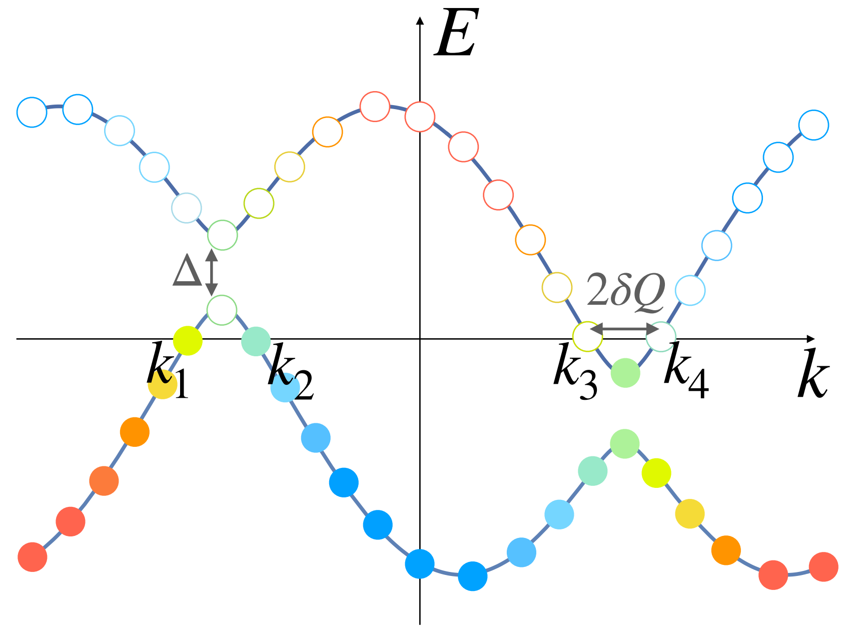
- This is in general incompatible with a trivial insulator.

TR-invariant Weyl semimetal

- Minimal model is obtained by breaking rotations and inversion in a type-I Dirac semimetal.



Lowest Landau levels



- Nonzero total momentum:

$$P_z = \pi \nu N_z \frac{B L_x L_y}{2\pi} \qquad \nu = \frac{2Q\delta Q d^2}{\pi^2}$$

Topological term

$$P_z = \pi \nu N_z \frac{B L_x L_y}{2\pi}$$

$$S = -i \frac{\nu}{2} \int z \wedge dz \wedge A$$

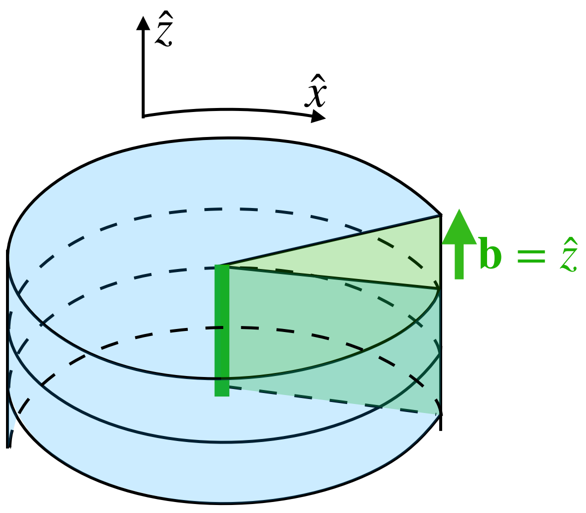
- Temporal component of z couples to the total momentum.

Charge on dislocation

$$S = -i\frac{\nu}{2}\int z \wedge dz \wedge A$$

- Charge on a screw dislocation along z:

$$Q_{U(1)} = \frac{\nu}{2}N_z$$



Unquantized anomalies and interactions

- Unquantized anomalies imply gapless modes with weak interactions.
- Does this remain true when the interactions are not weak?
- Interactions may always simply tune the coefficient of the anomaly term to a trivial value, need to fix the coefficient to make this question nontrivial.
- Can we gap out Weyl nodes in magnetic Weyl semimetal while keeping the Hall conductivity fixed?

3D Fractional Quantum Hall Effect

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Fractional Quantum Hall Effect in Weyl Semimetals

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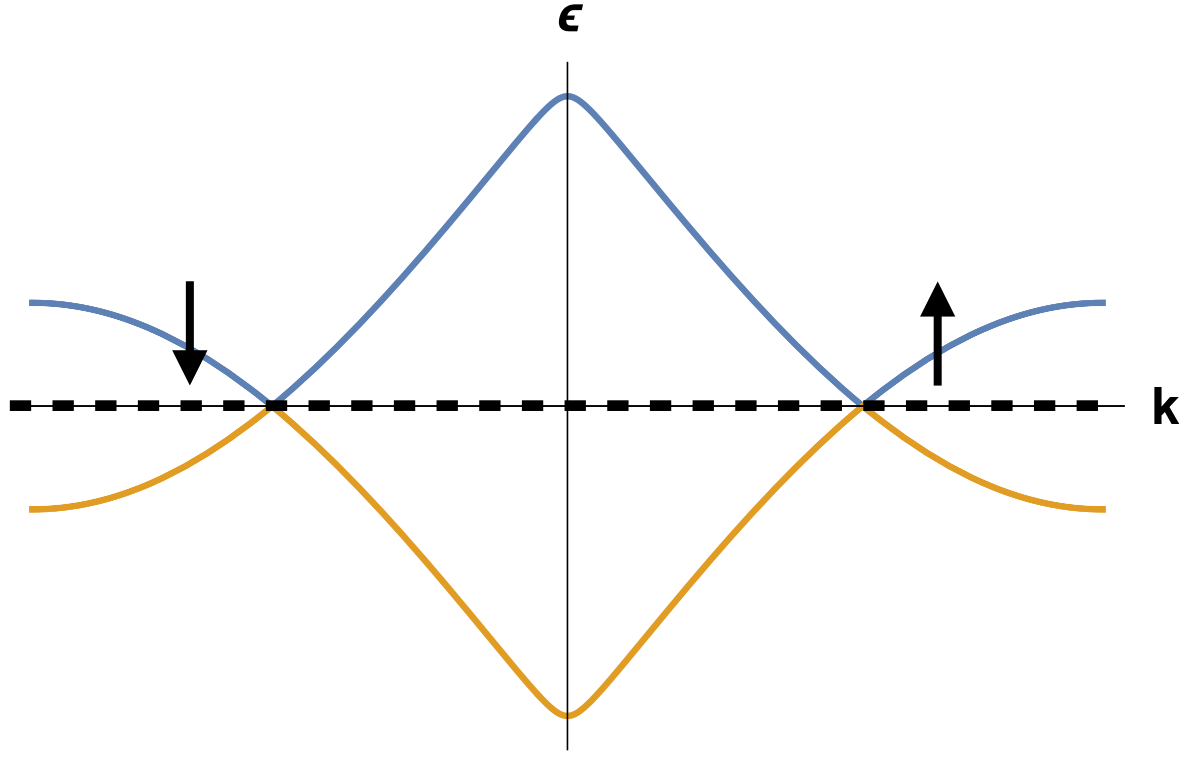
- Chiral anomaly turns out to be consistent with a gapped fractionalized state at one particular value of the Weyl node separation.

Vortex condensation

- Induce fully gapped superconductivity in Weyl semimetal.
- Destroy SC coherence by condensing vortices while keeping the pairing gap: this produces an insulator (superconductor to insulator transition).
- Chiral anomaly places strong restrictions on the procedure and prohibits a simple insulator, has to have topological order.

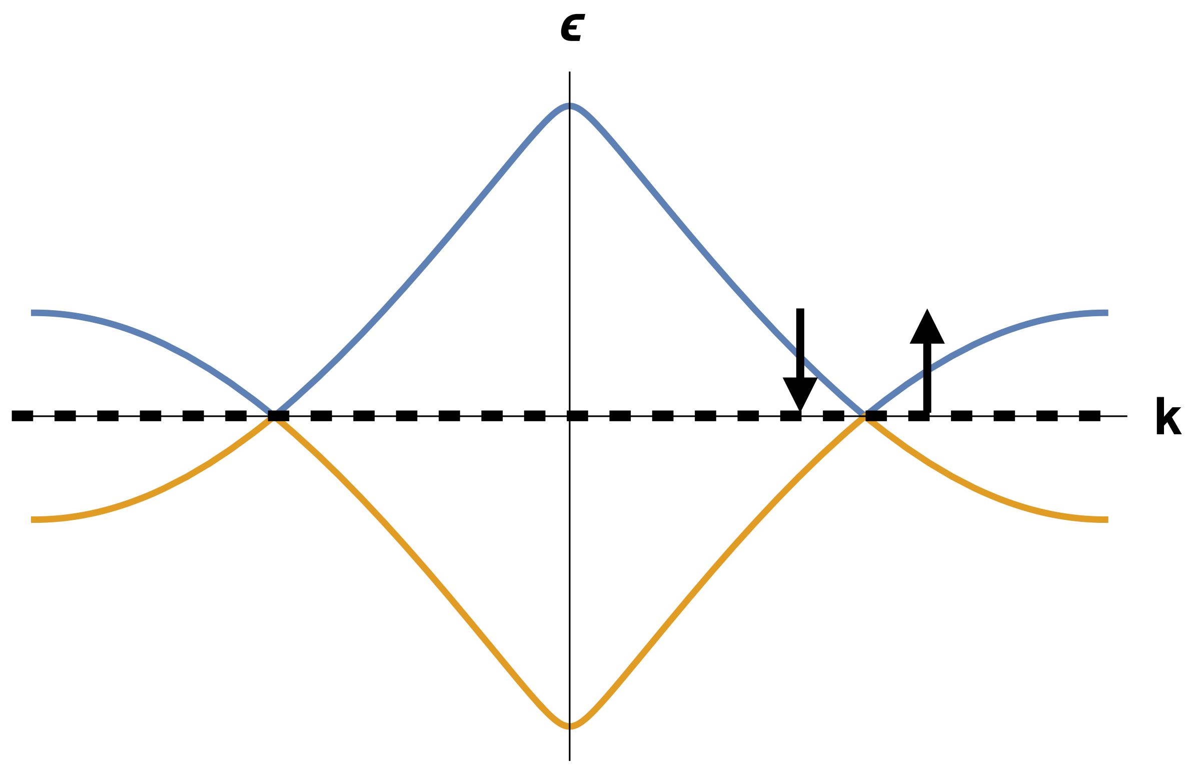
Weyl superconductor

- BCS: pairing k and -k states, i.e. internodal pairing.



Weyl superconductor

- FFLO (Fulde-Ferrell-Larkin-Ovchinnikov): pairing states on the opposite side of each Weyl point, i.e. intranodal pairing.



BCS pairing

- Weak BCS pairing can not open a gap, since the two chiralities are not mixed by the pairing term:

$$H = v_F \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}} \tau^z \boldsymbol{\sigma} \cdot \mathbf{k} c_{\mathbf{k}} + \Delta \sum_{\mathbf{k}} (c^\dagger_{\mathbf{k}R} i\sigma^y c^\dagger_{-\mathbf{k}L} + h.c.)$$

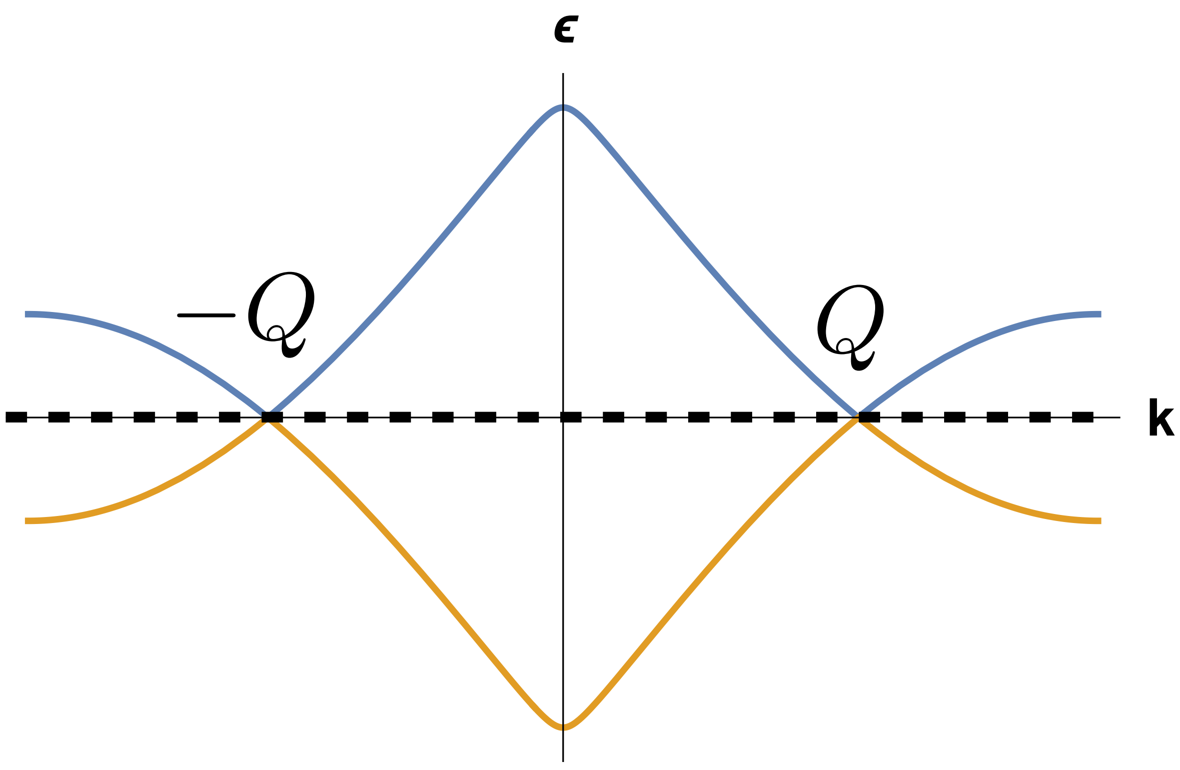
$$\psi_{\mathbf{k}} = (c_{\mathbf{k}R\uparrow}, c_{\mathbf{k}R\downarrow}, c^\dagger_{-\mathbf{k}L\downarrow}, -c^\dagger_{-\mathbf{k}L\uparrow})$$

$$H = \sum_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} (v_F \boldsymbol{\sigma} \cdot \mathbf{k} + \Delta \tau^x) \psi_{\mathbf{k}}$$

Meng & Balents   Bednik, Zyuzin, AAB

FFLO pairing

- FFLO does open a gap, but breaks translational symmetry:

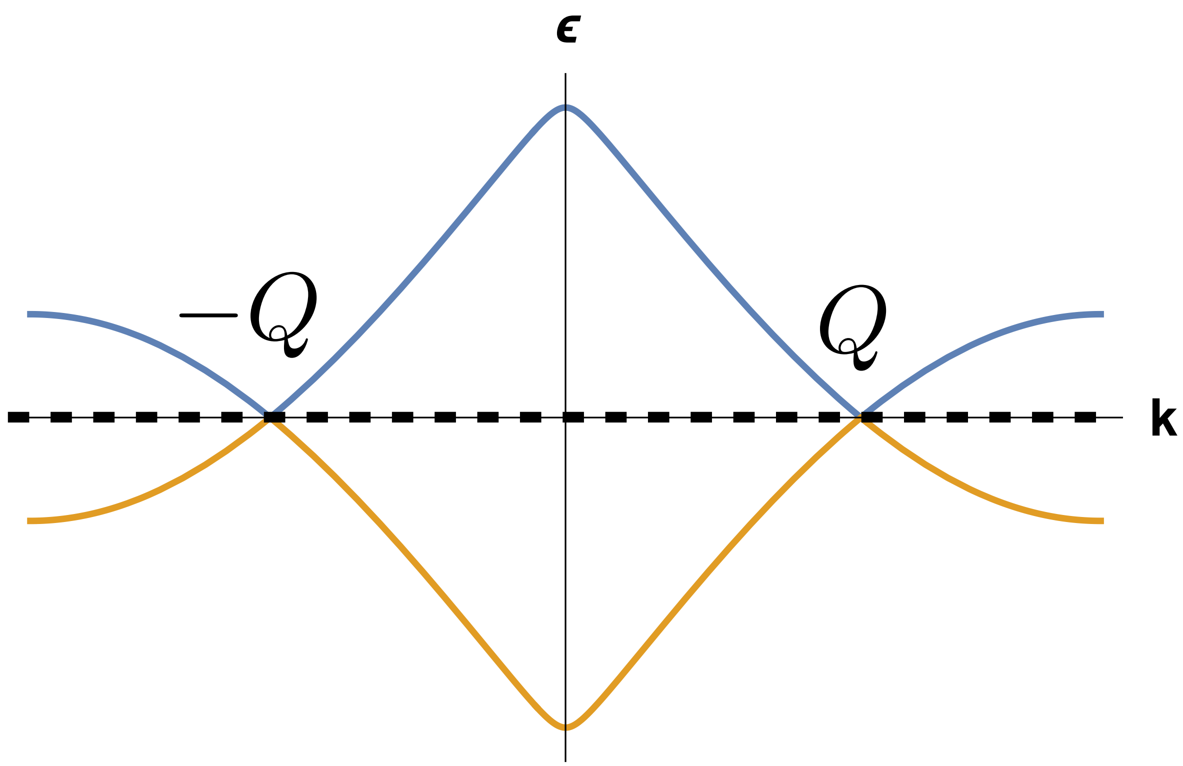


$\Delta(\mathbf{Q}) \sim \sum_{\mathbf{k}} \langle c^\dagger_{\mathbf{Q}+\mathbf{k}} c^\dagger_{\mathbf{Q}-\mathbf{k}} \rangle$

carries momentum 2Q.

$\varrho(\mathbf{Q}) \sim \Delta^*(-\mathbf{Q})\Delta(\mathbf{Q})$ carries momentum 4Q.

FFLO pairing



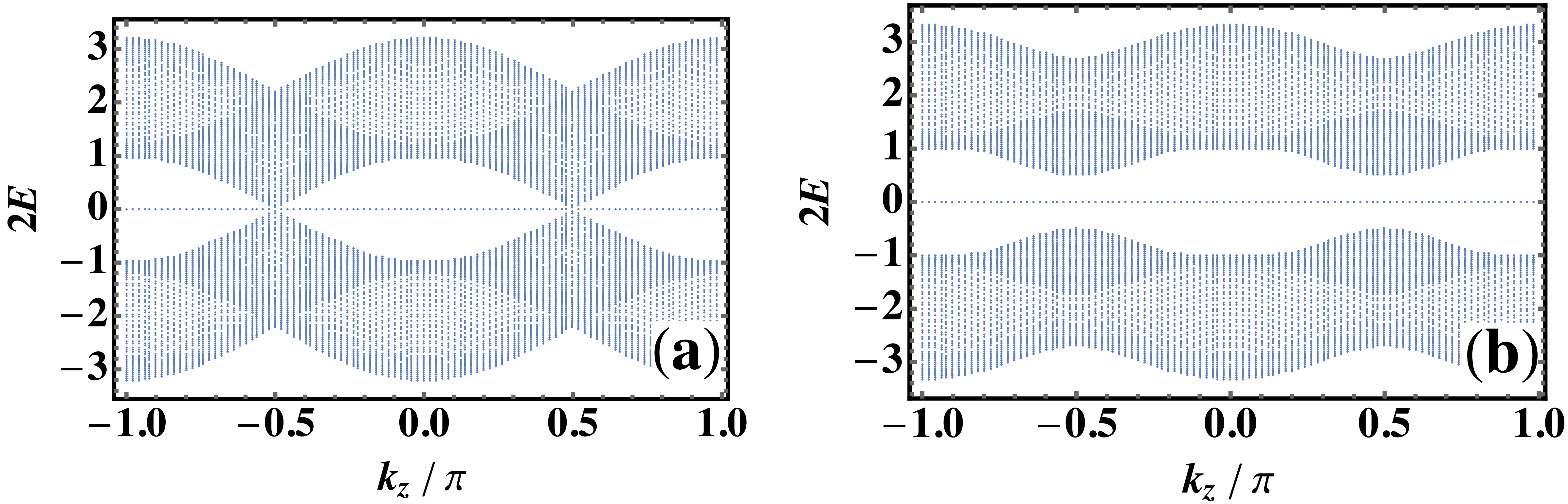
$$\varrho(\mathbf{Q}) \sim \Delta^*(-\mathbf{Q})\Delta(\mathbf{Q})$$

carries momentum 4Q.

- This breaks translational symmetry, unless $\mathbf{Q} = \mathbf{G}/4$
- In other words, FFLO does not break translational symmetry when Weyl node separation is exactly half the reciprocal lattice vector.

Majorana surface state

- Fermi arc becomes Majorana surface mode, which occupies twice the momentum interval of the Fermi arc, i.e. 4Q.

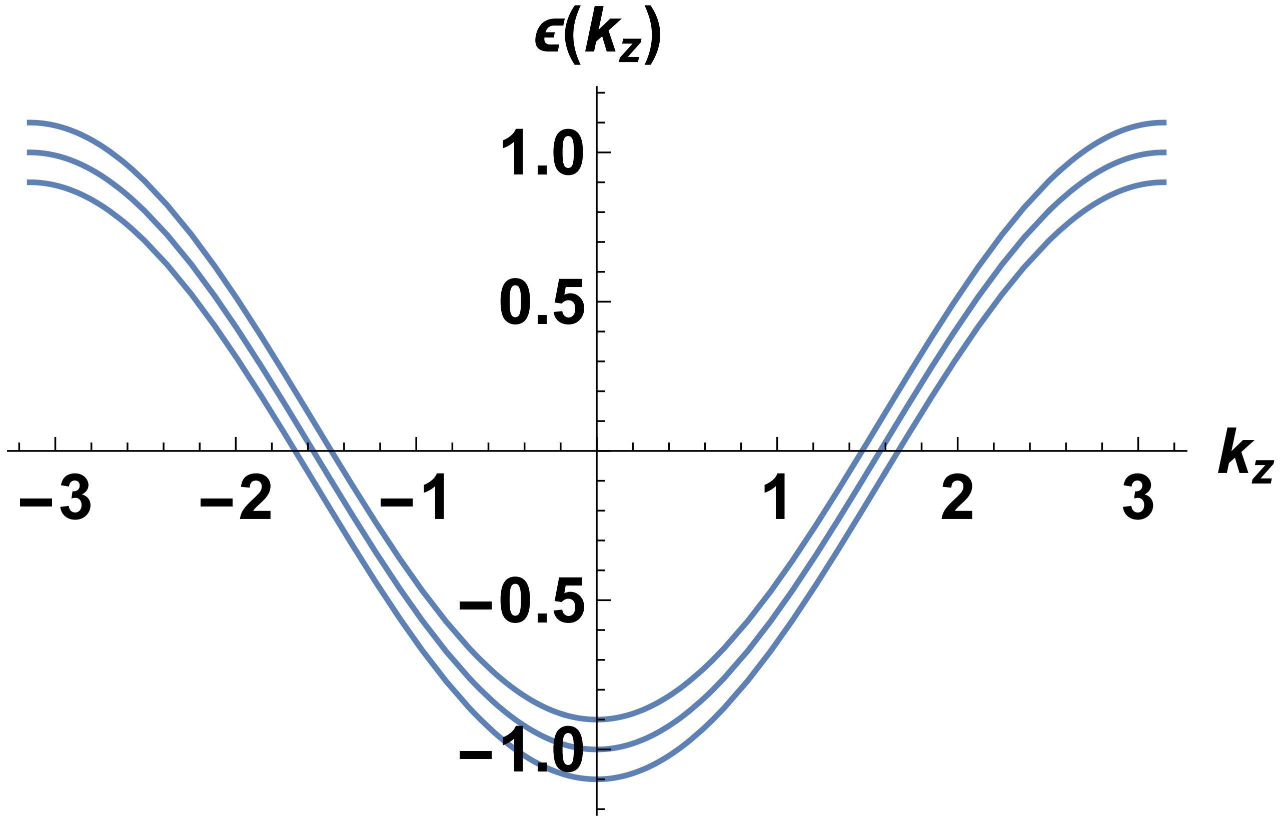


$$\kappa_{xy} = \sigma_{xy}\left(\frac{\pi^2 k_B^2 T}{3}\right) = \frac{1}{4\pi}\left(\frac{\pi^2 k_B^2 T}{3}\right)$$

Vortex condensation in FFLO state

- n-fold vortex (Φ=nhc/2e) in FFLO paired state: get n chiral Majorana modes in the vortex core.

$$\epsilon_p(k_z) = \epsilon_F\left(1 - \frac{2p}{n+1}\right) + v_F k_z. \qquad p = 1, \ldots, n.$$

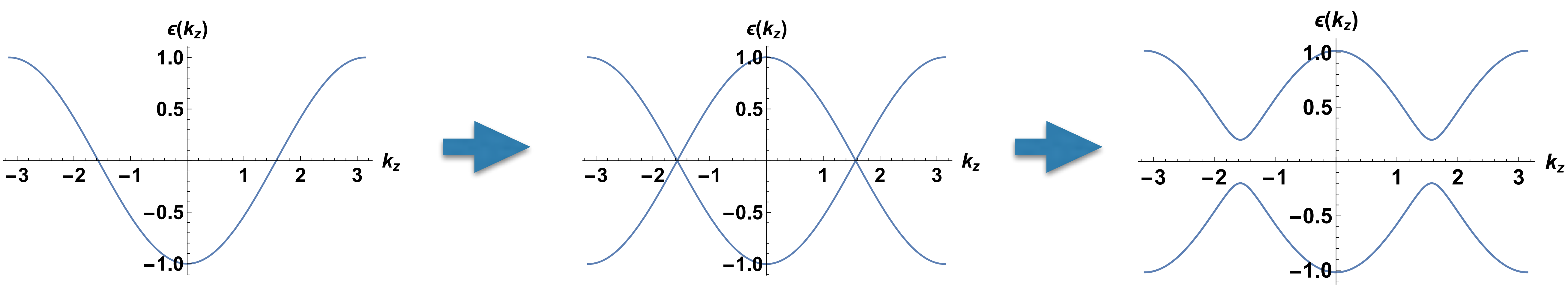


Callan & Harvey

Jackiw & Rossi

Vortex condensation in FFLO state

- Any even number 2n of Majorana vortex modes may be combined into n 1D Weyl fermion modes, which are gapped out by pairing:



Vortex condensation in FFLO state

- Any even number 2n of Majorana vortex modes may be combined into n 1D Weyl fermion modes, which are gapped out by pairing:

$$H = v_F \sum_{k_z} [k_z c^\dagger_{k_z} \tau^z c_{k_z} + \Delta (c^\dagger_{k_z} i\tau^y c^\dagger_{-k_z} + \mathrm{h.c.})/2]$$

- An odd number of Majorana modes can not be eliminated without breaking translational symmetry, thus a fundamental SC vortex may not be condensed.

$$\Phi = \frac{hc}{2e} = \pi \qquad \hbar = c = e = 1$$

Vortex condensation in FFLO state

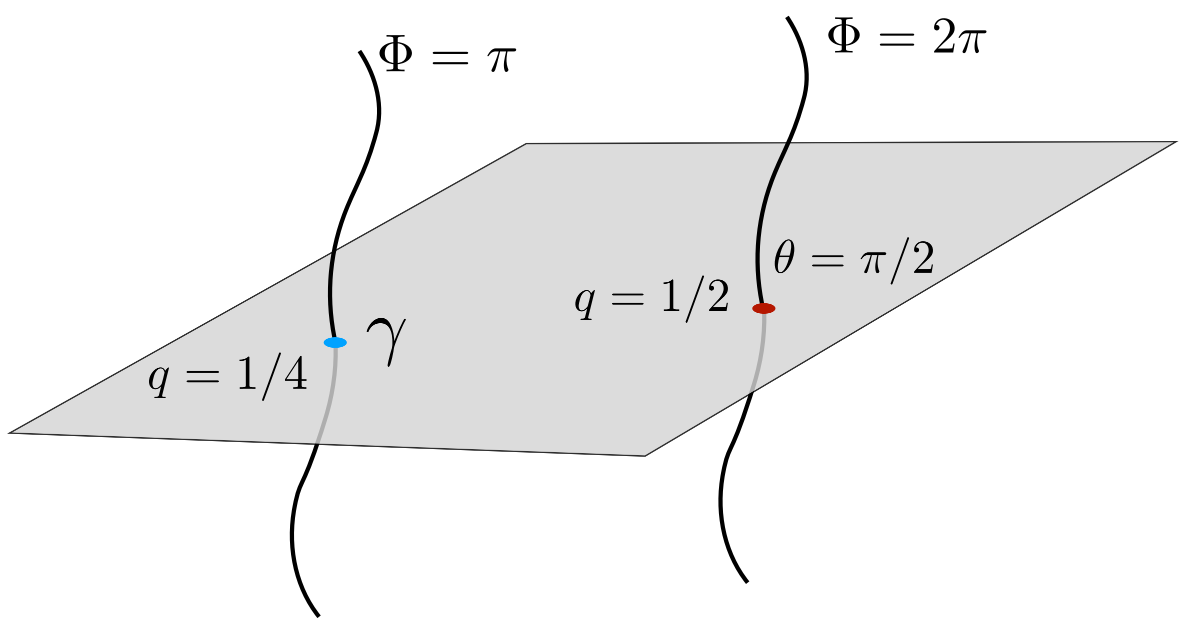
- A double vortex does not have Majorana modes, but may still not be condensed.

- This follows from the fact that the insulating state we want to obtain must preserve the chiral anomaly, i.e. must have a Hall conductivity of half conductivity quantum per atomic plane:

$$\sigma_{xy} = \frac{1}{2\pi}\frac{2Q}{2\pi} = \frac{1}{4\pi}$$

Vortex condensation in FFLO state

- A vortex will induce a charge when intersecting an atomic plane:



- A pair of such charges will have semion exchange statistics.

$$\theta = 2\pi^2 \sigma_{xy} = \frac{\pi}{2}$$

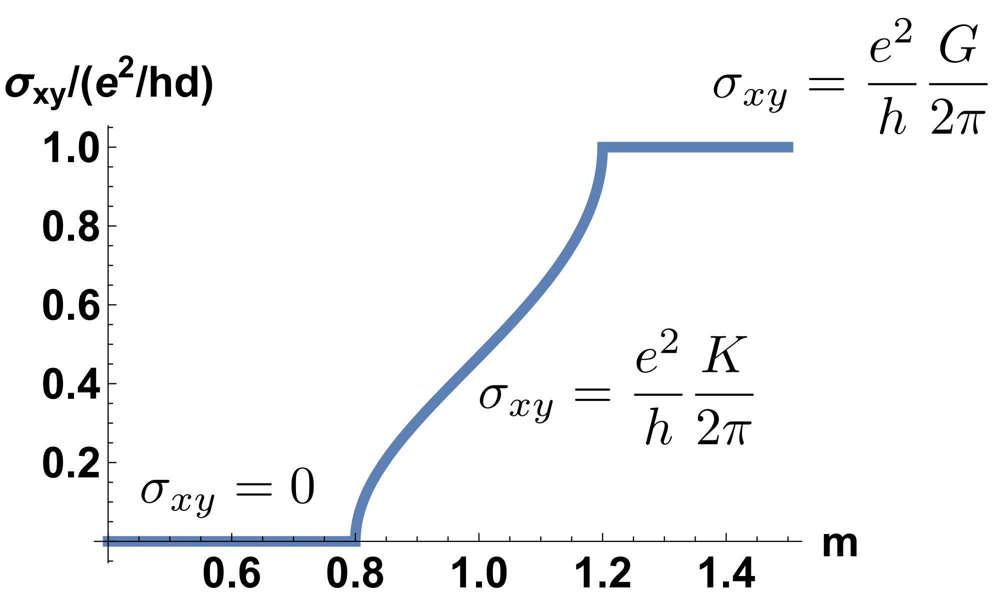
Vortex condensation in FFLO state

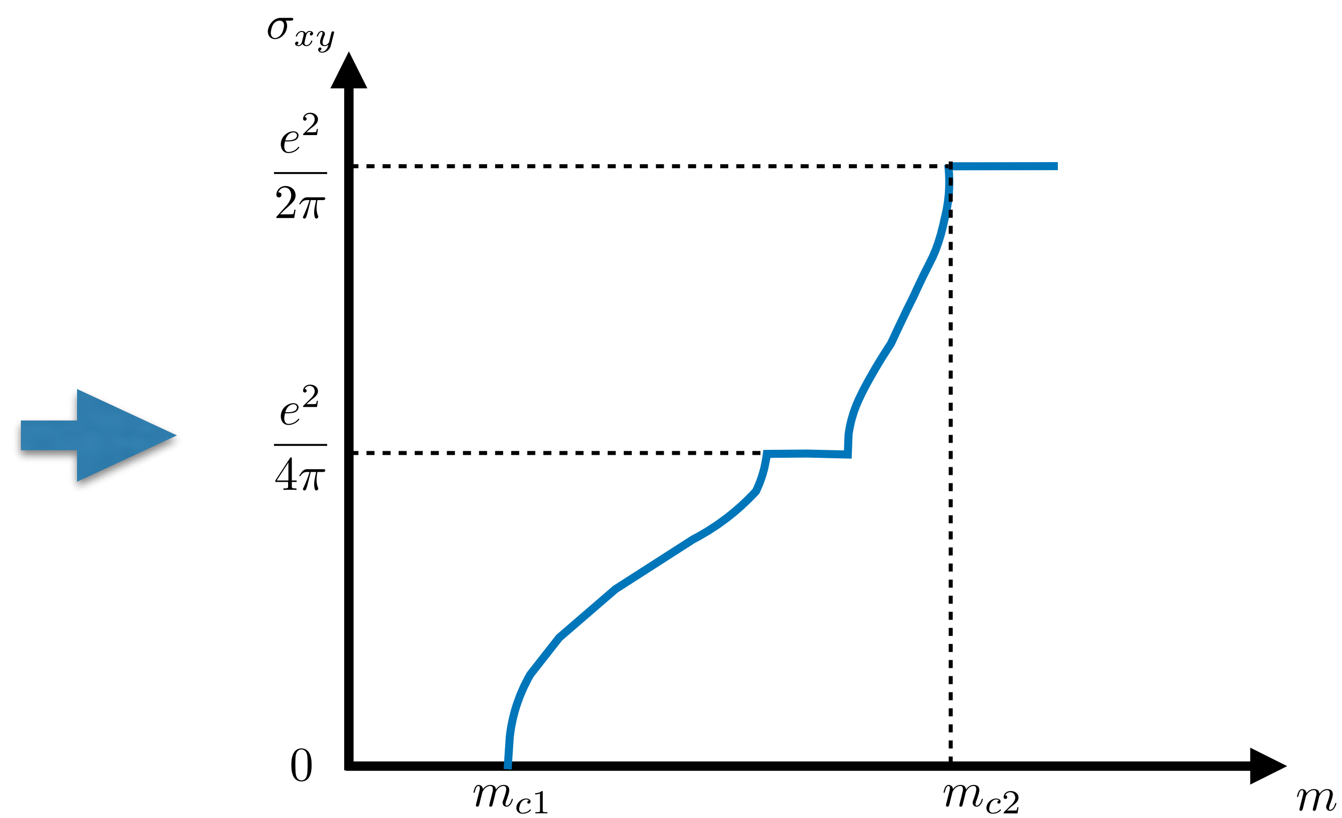
- Following the same logic, quadruple vortices have bosonic statistics and thus may be condensed without breaking any symmetries.

- This is an insulating state that preserves the chiral anomaly and does not break any symmetries.

$$\sigma_{xy} = \frac{1}{2\pi}\frac{2Q}{2\pi} = \frac{1}{4\pi} \qquad \kappa_{xy} = \sigma_{xy}\left(\frac{\pi^2 k_B^2 T}{3}\right) = \frac{1}{4\pi}\left(\frac{\pi^2 k_B^2 T}{3}\right)$$

Nontrivial generalization of FQHE to 3D





- In the presence of interactions, smooth evolution of the Hall conductivity with the magnetization in a Weyl semimetal may be interrupted by a half-quantized plateau.

BF theory of the 3D FQHE

- 2D FQHE: Chern-Simons theory.

Odd-denominator Laughlin state $\nu = \frac{1}{2q+1}$

$$\mathcal{L} = i\frac{2q+1}{4\pi}\epsilon_{\mu\nu\lambda}b_\mu\partial_\nu b_\lambda + \frac{ie}{2\pi}\epsilon_{\mu\nu\lambda}A_\mu\partial_\nu b_\lambda + ib_\mu j_\mu$$

- Excitations are quasiparticles (vortices), which carry fractional charge and fractional statistics:

$$Q = \frac{e}{2q+1} \qquad \theta = \frac{\pi}{2q+1}$$

BF theory of the 3D FQHE

$$\mathcal{L} = \mathcal{L}_f(-a_\mu) + \frac{i}{2\pi}(A_\mu + a_\mu + 2c_\mu)\epsilon_{\mu\nu\lambda\rho}\partial_\nu b_{\lambda\rho} - \frac{2i}{4\pi}\epsilon_{z\mu\nu\lambda}c_\mu\partial_\nu c_\lambda + ib_{\mu\nu}j_{\mu\nu} + ic_\mu j_\mu$$

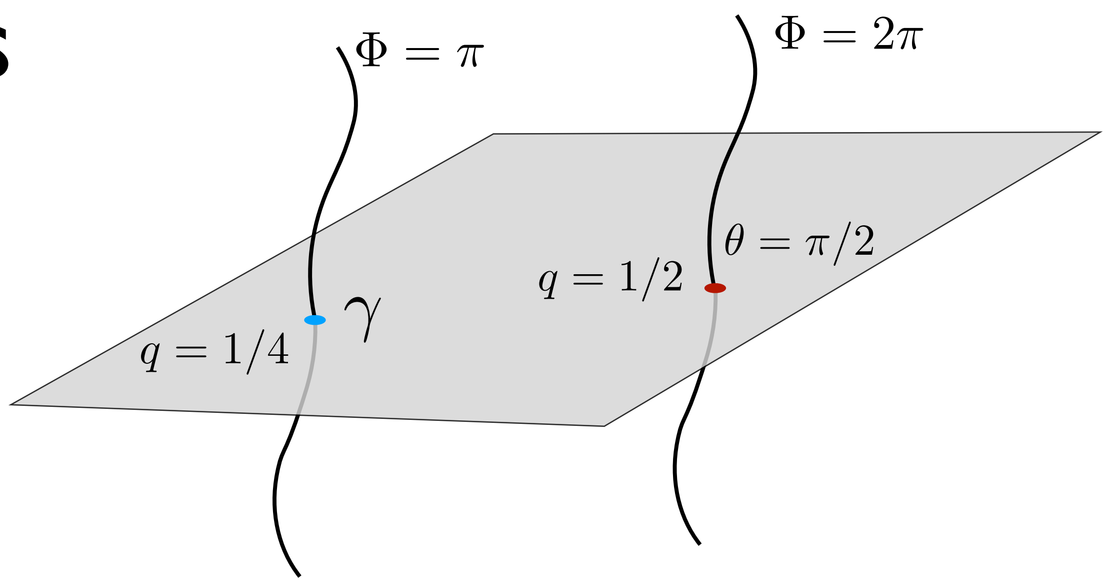
- Neutral fermions (couple to a_μ).
- Charged bosons (couple to c_μ).
- Vortex loops (couple to $b_{\mu\nu}$).
- a_μ is a Z_2 gauge field, while c_μ is a Z_4 gauge field.

Thakurathi & AAB

BF theory of the 3D FQHE

$$\mathcal{L} = \mathcal{L}_f(-a_\mu) + \frac{i}{2\pi}(A_\mu + a_\mu + 2c_\mu)\epsilon_{\mu\nu\lambda\rho}\partial_\nu b_{\lambda\rho} - \frac{2i}{4\pi}\epsilon_{z\mu\nu\lambda}c_\mu\partial_\nu c_\lambda + ib_{\mu\nu}j_{\mu\nu} + ic_\mu j_\mu$$

- Intersections of vortex loops with atomic planes are anyons.



Conclusions

- Topological semimetals may be characterized by unquantized (i.e. having tunable coefficients) anomalies.

- The anomalies may be expressed as topological terms, involving electromagnetic as well as crystal symmetry gauge fields.

- These topological terms describe fractional electric charges, induced on symmetry defects, such as flux lines, dislocations and disclinations.

- Chiral anomaly in a magnetic Weyl semimetal is consistent with a fully gapped fractionalized insulator, which is a nontrivial generalization of the fractional quantum Hall liquid to three dimensions.