Research Summary

Over the past decades, knots have shown themselves to be at the intersection of several interesting fields of mathematics, as well as the interface with the other scientific fields. Beyond their intrinsic appeal as a remarkably accessible portion of mathematics, they have appeared naturally in the study of representation theory, 3-manifolds, quantum field theory, category theory, and quantum computing. Thus knots, while interesting on their own, are also an excellent proofing ground for ideas from other parts of mathematics.

Despite the fact that the problem can be explained to a child with a piece of string, the question of distinguishing knots is still a rather difficult one, and one being investigated by an increasingly large zoo of knot invariants. One particularly exciting development in this field has been the appearance of knot invariants of a homological nature, that is, invariants given by graded vector spaces appearing as the homology of a chain complex. These invariants are interesting not just because of their potential strength for distinguishing knots, but also because at least some are functorial. Given the extraordinary power that homological invariants have shown in fields as diverse as topology, number theory, representation theory, algebraic geometry and commutative algebra, there's cause to be sanguine about the possibility of using them in knot theory.

My personal research is in connecting these invariants to geometric representation theory. All currently known homological knot invariants correspond to previously understood polynomial invariants of knots, constructed from the representation theory of quantum groups. But only some of these polynomial invariants have been lifted to the homological level. This task has only been carried out for those representations which we best understand combinatorially. A more general definition will probably require a detailed understanding of categories corresponding to tensor products of quantum group representations.

I plan to approach this problem from the perspective of the geometry of quiver varieties. Work of Lusztig, Nakajima, and Zheng has already shown that there is an intimate connection between quiver varieties and quantum groups, and has produced a candidate category for giving homological counterparts to all quantum invariants of knots. My goal is to define said homological invariants, investigate their properties (including functoriality), and show whether they're related to categorifications of quantum groups given by Khovanov and Lauda.

These categories are in fact a special case of a more general construction of categories associated to symplectic singularities. Some of these are of great interest to geometric representation theorists when understood as representations of noncommutative algebras such as universal enveloping algebras, Cherednik algebras, and W-algebras. I'm interested in investigating general properties of these categories, and particular cases such as hypertoric singularities and instantons on ALE spaces.

I've been working with T. Braden, A. Licata, and N. Proudfoot on investigating these examples, and on formulating a notion of duality for symplectic singularities which is manifested as Koszul duality for these categories. This duality seems to be connected to the mirror symmetry of 3-dimensional gauge theories in physics (which is rather different in flavor and much less thoroughly investigated than the mirror symmetry between 2-dimensional sigma-models more familiar to mathematicians), and thus presents a very fertile area for interactions between mathematics and physics.