Theorem 3 (Hernandez-Oya, 2018)**.** *The conjecture of* [H] *is true in type B : a Kazhdan-Lusztig algorithm gives the dimensions and characters of simple finite-dimensional modules.*

References

[CP] V. Chari and A. Pressley, *Quantum affine algebras and affine Hecke algebras*, Pacific J. Math. **174** (1996), no. 2, 295–326.

[FR] E. Frenkel and N. Reshetikhin, *The q-characters of representations of quantum affine algebras and deformations of W-Algebras*, Contemp. Math. **248** (1999), 163–205.

[H] D. Hernandez, *Algebraic approach to q,t-characters*, Adv. Math. **187**, 1–52 (2004).

[HL1] D. Hernandez and B. Leclerc *Cluster algebras and quantum affine algebras*, Duke Math. J. **154** (2010), no. 2, 265–341.

[HL2] D. Hernandez and B. Leclerc, *Quantum Grothendieck rings and derived Hall algebras*, J. Reine Angew. Math. **701**, 77–126 (2015).

[HL3] D. Hernandez and B. Leclerc, *A cluster algebra approach to q-characters of Kirillov-Reshetikhin modules*, J. Eur. Math. Soc. **18**, no. 5, 1113–1159 (2016).

[HO] D. Hernandez and H. Oya, *Quantum Grothendieck ring isomorphisms, cluster algebras and Kazhdan-Lusztig algorithm*, Preprint arXiv:1803.06754

[KKO] M. Kashiwara, M. Kim and S.-j. Oh, *Monoidal categories of modules over quantum affine algebras of type A and B*, Preprint arXiv:1710.06627

[N] H. Nakajima, *Quiver varieties and t-analogs of q-characters of quantum affine algebra*, Ann. of Math. (2) **160**, no. 3, 1057–1097 (2004).

[VV] M. Varagnolo and E. Vasserot, *Perverse sheaves and quantum Grothendieck rings*, Studies in memory of Issai Schur (Chevaleret/Rehovot, 2000), Progr. Math. **210**, Birkhäuser Boston, Boston, MA, 345–365 (2003).

Supported by ERC with Grant Agreement number 647353 Qaffine.

The classification of Gelfand-Tsetlin modules and the Braverman-Finkelberg-Nakajima construction

Ben Webster

(joint work with Oded Yacobi, Alex Weekes)

One very challenging problem in the representation theory of the Lie algebra $\mathfrak{gl}_n$ is the classification of Gelfand-Tsetlin modules, that is, the finitely generated modules where the Gelfand-Tsetlin subalgebra Γ generated by the centers of the universal enveloping algebras $U(\mathfrak{gl}_1) \subset U(\mathfrak{gl}_2) \subset \cdots \subset U(\mathfrak{gl}_n)$ acts locally finitely. See [FGR, H] for a more general discussion of this problem.

The heart of our approach is the use of the generalized weight functors

$$W_{\mathfrak{m}}(M) = \{m \in M \mid \mathfrak{m}^N m = 0 \ \forall N \gg 0\}$$

for the different maximal ideals $\mathfrak{m} \in \mathrm{MaxSpec}(\Gamma)$. These functors are exact, and for any Gelfand-Tsetlin module $M \cong \bigoplus_{\mathfrak{m}\in\mathrm{MaxSpec}(\Gamma)} W_{\mathfrak{m}}(M)$. On very general grounds, the category of Gelfand-Tsetlin modules is thus controlled by the category whose objects are these functors, with morphisms given by natural transformations.

This category becomes much easier to analyze when we realize $U(\mathfrak{gl}_n)$ as a quantum Coulomb branch, in the sense of [BFN]. This allows us to identify the space of natural transformations between two weight functors as the homology of a Steinberg type space; in fact, when we consider the endomorphisms of an appropriate sum of weight spaces, it is precisely a completed weighted KLR algebra as defined in [W2], following the approach of [W1, KTWWY]. This fact allows us to complete the desired classification, and answer many questions about the structure of simple Gelfand-Tsetlin modules, by giving a finite dimensional algebra whose simple representations are in bijection with Gelfand-Tsetlin modules of a fixed weight. In particular, we identify the set of simple integrable Gelfand-Tsetlin modules with fixed central character with the dual canonical basis of the zero weight space of a tensor product of $U(\mathfrak{n}) \otimes (\mathbb{C}^n)^{\otimes n}$ of the inversal enveloping algebra of lower triangular matrices $\mathfrak{n}$ with n copies of the standard representation of $\mathfrak{sl}_n$. The other weight spaces of this tensor product correspond to similar module categories for W-algebras or orthogonal Gelfand-Tsetlin algebras [M].

This same approach can be applied to other quantized Coulomb branches, such as rational Cherednik algebras, as well as other principal Galois orders (as introduced in [H]). However, such algebras which are not Coulomb branches will require new calculations.

References

[BFN] Alexander Braverman, Michael Finkelberg, and Hiraku Nakajima, *Towards a mathematical definition of Coulomb branches of* 3*-dimensional* $\mathcal{N} = 4$ *gauge theories, II*, Preprint arXiv:1601.03586.

[FGR] Vyacheslav Futorny, Dimitar Grantcharov, and Luis Enrique Ramirez, *Singular GelfandTsetlin modules of* $\mathfrak{gl}(n)$. Adv. in Math. **290** (2016), 453–482.

[FO] Vyacheslav Futorny and Serge Ovsienko, *Fibers of characters in Gelfand-Tsetlin categories*, Trans. Amer. Math. Soc. **366** (2014), no. 8, 4173–4208. MR 3206456

[H] Jonas T. Hartwig, *Principal Galois orders and Gelfand-Zeitlin modules*, Preprint arXiv:1710.04186.

[KTWWY] Joel Kamnitzer, Peter Tingley, Ben Webster, Alex Weekes, and Oded Yacobi, *On category* $\mathcal{O}$ *for affine Grassmannian slices and categorified tensor products*, Preprint arXiv:1806.07519.

[M] Volodymyr Mazorchuk, *Orthogonal Gelfand-Zetlin Algebras I*, Contributions to Algebra and Geometry **40** (1999), no. 2, 399–415.

[MV] Ian M. Musson and Michel Van den Bergh, *Invariants under tori of rings of differential operators and related topics*, Mem. Amer. Math. Soc. **136** (1998), no. 650, viii+85.

[W1] Ben Webster, *Koszul duality between Higgs and Coulomb categories* $\mathcal{O}$, Preprint arXiv:1611.06541.

[W2] ———, *Weighted Khovanov-Lauda-Rouquier algebras*, Preprint arXiv:1209.2463.