Pure Math 945
Combinatorial Representation Theory

Below are some topics that might be interesting for your final project/talk. The expectation is that you will give a 15 minute presentation and write a paper of about 5 pages (so keep in mind that the actual amount of content will be very small!). These are only suggestions, and you are free to consider other topics, but either way, pleae consult with me when choosing a topic (for example, so I can coordinate people working on the same topic).

1) The representation theory of the alternating group: the representations of the alternating group are connected to those of the symmetric group by induction and restriction, so we can describe the representations of the alternating group (see this article, also by CST).

2) The hook length formula: this is a formula for the dimension of irreps of S_n. (Sagan, "The Symmetric Group" 3.10).

3) Jeu de taquin: this is a "game" that lets us relate tableau of different skew shapes. (Sagan, "The Symmetric Group" 3.7).

4) The Robinson-Schensted correspondence: this is a bijection between pairs of standard tableaux and permutations, i.e. realizing the fact that order of the symmetric group is the sum of the squares of the dimensions of its representations (Sagan, "The Symmetric Group" 3.1).

5) The Littlewood-Richardson rule (notes of van Leeuwen)

6) Symmetric functions and the cohomology of the Grassmannian (you can look at these notes or Fulton's "Young Tableaux")

7) Schur-Weyl duality (CST "Representation Theory of the Symmetric Groups" Section 8.2).

8) Partition algebras. These play a dual role from S_n in its action on tensor products of its reflection representation (CST "Representation Theory of the Symmetric Groups" Section 8.3).

9) Cellular algebras (there are notes from a course by Xi).

10) The relating modular and ordinary representations of finite groups (Chapters 14-17 of Serre and Chapter 9 of Webb).

11) Block theory for modular representations (Chapter 12 of Webb).

12) Brauer characters (in general and for S_n) (Chapter 18 of Serre and Chapter 10 of Webb).

13) Ariki-Koike and cyclotomic Schur algebras (see the survey of Mathas).

14) Category $\mathcal{O}$ and degenerate affine Hecke algebras (see the original paper of Arikawa and Suzuki) Mazorchuk "Lectures on Algebraic categorification.").

15) Broué's conjecture for symmetric groups (see Chapter 14 of Mazorchuk "Lectures on Algebraic categorification.").

16) Crystals from Hecke algebras/KLR algebras (the original paper of Lauda and Vazirani).

17) Cherednik algebras and the KZ functor (these notes are a good starting point; these by Etingof are much more detailed).